Sharpened Sharpes

Jon Ingersoll

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Example:

Manager 1: average returns of 25%
Manager 2: average returns of 5%

Who was better?

Manager 1

but insufficient information

Risk Adjustment

Why do we adjust for risk?

Riskier assets earn more
Levering increases average return

Risk Adjustment

$$S = \frac{\text{performance}}{\text{risk}}$$

$$S = (\text{performance}) - \begin{pmatrix} \text{adjustment} \\ \text{for risk} \end{pmatrix}$$

Sharpe Ratio

$$S_0 = \frac{\text{E[Excess return]}}{\sqrt{\text{Var[returns]}}}$$

One of the first

Still one of the commonly used

Sharpe Ratio for Example

Suppose $r_f = 10\%$

Then in example average excess returns

Manager 1: 15%
Manager 2:   -5%

Manager 1 has higher Sharpe regardless of measured risk

Failure of Sharpe Measure

Manager 1 rates of returns:
+100% -50%

Manager 2 rates of return
always 5%

Manager 2 is clearly better.

Experiment: Maximize Sharpe

Thought Experiment:

Complete market
Manager maximizes Sharpe ratio

What do we get?

Experiment: Maximize Sharpe

Choose excess returns x_i to

$$\operatorname*{Max}_{x_i} \frac{\bar{x}}{\left[\sum p_i (x_i - \bar{x})^2\right]^{1/2}}$$

subject to $\sum p_i x_i = \bar{x}$ and $\sum \hat{p}_i x_i = 0$

$\hat{p}_i$ is the risk-neutral probability

p_i is the true probability

Experiment Results

$$x_i^* = \bar{x}\left[1 + \frac{1 - \hat{p}_i / p_i}{\sum \hat{p}_i^2 / p_i - 1}\right]$$

Hold more when $\hat{p}_i / p_i$ is small.

i.e., hold more in good states

x_i has an upper bound

no lower bound not even –100%

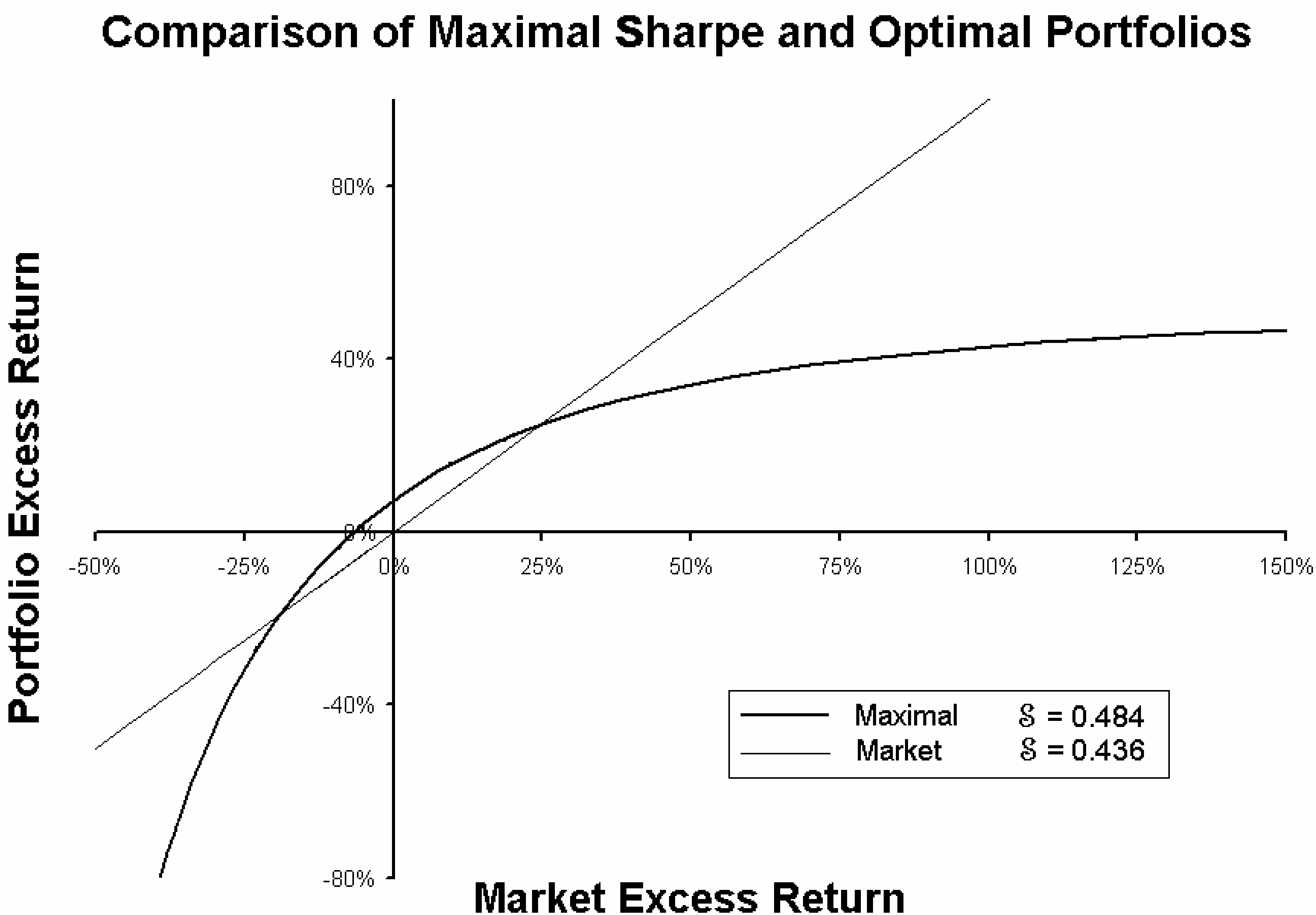
Experiment Results

How “good” can you be?

$$S_0^* = \left(\sum \hat{p}_i^2 / p_i - 1 \right)^{1/2} = \text{StdDev}\left[\hat{p}_i / p_i \right]$$

The more risk aversion
and the more risk

The higher can we push 💧$_0$


Comparison of Maximal Sharpe and Optimal Portfolios
Portfolio Excess Return
80%
40%
0%
-40%
-80%
-50%
-25%
0%
25%
50%
75%
100%
125%
150%
Maximal
Market
𝕊 = 0.484
𝕊 = 0.436
Market Excess Return

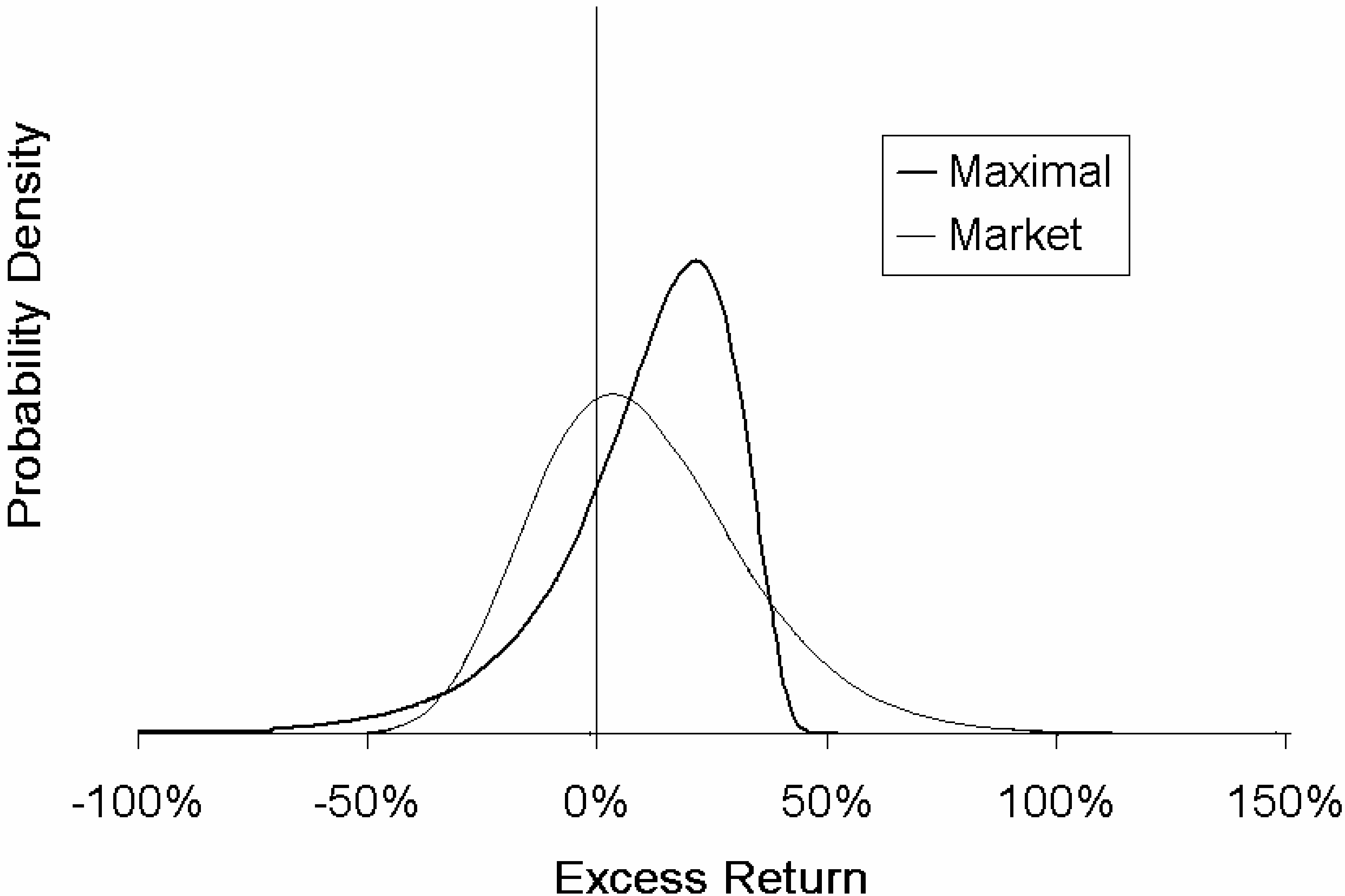
How Big an Effect

$\mu - r$ = 10% σ =15% (continuous)

Market Sharpe = 0.436
Maximal Sharpe = 0.484
11% improvement

MSRP beats market 65% of the time
all in the middle ranges

Distributions of Portfolio Returns



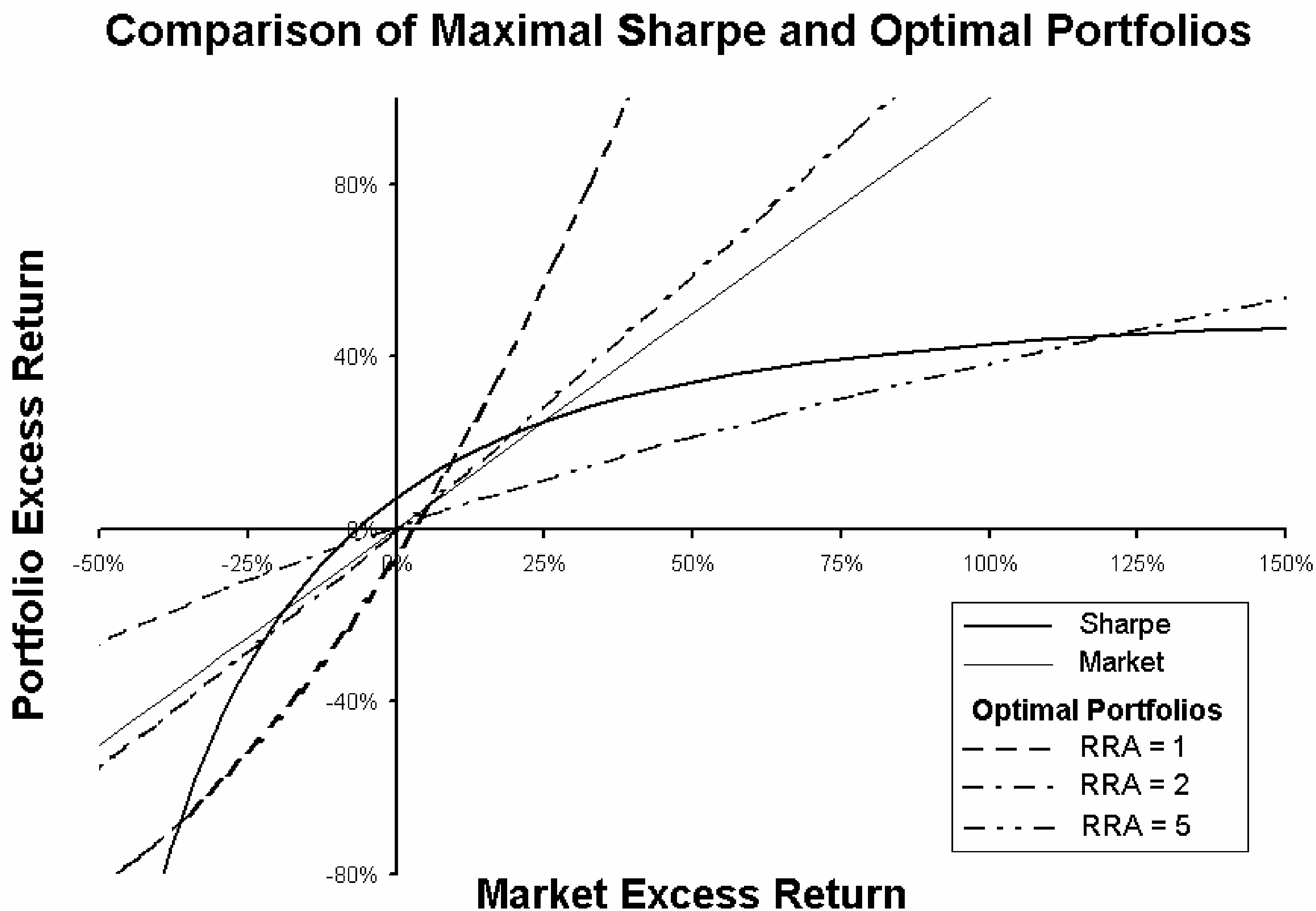
What Then Are Good Portfolios?

Standard Economic Theory says:

Maximize expected utility

$$\text{Max}\,\mathrm{E}[u(1 + r_f + \tilde{x})]$$

Common utility functions are
Log, exponential, power



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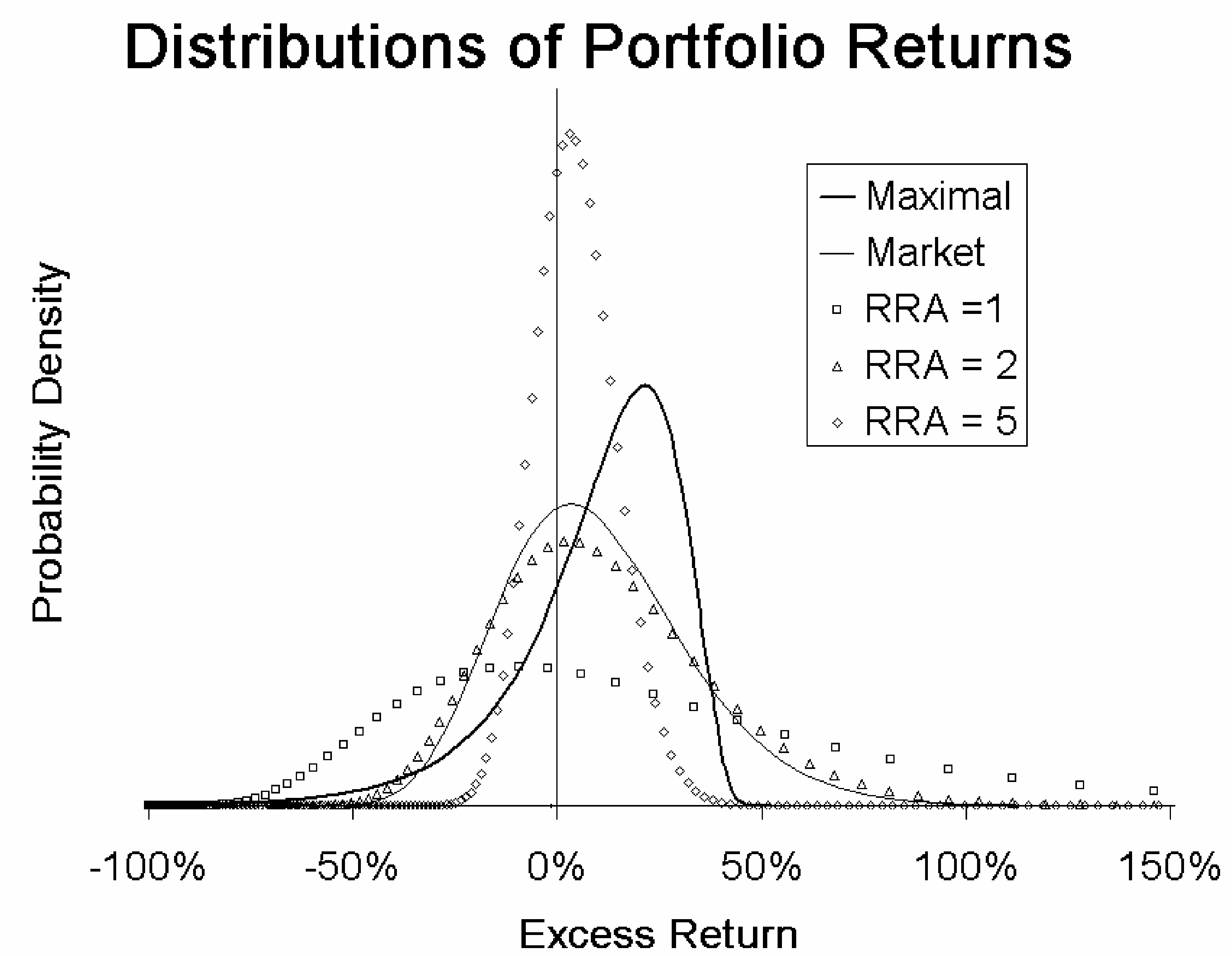


Sharpened Sharpes

How Big an Effect?

	Sharpe Ratio	% time beaten by MSRP
MSRP:	0.484	0
Market:	0.436	65%
RRA = 1:	0.397	57%
RRA = 2:	0.432	65%
RRA = 5:	0.450	84%

An Academic Problem?

No.

Selling 5% out-of-the-money puts and calls can capture 95% of the Sharpe ratio improvement.

Even just throwing away very good outcomes helps

Special Circumstances for Hedge Funds

The high-water mark incentive fee serves to “throw away” very high outcomes.

Accounting can “smooth” those extreme outcomes to next bad period.

Cause of Problem

Primary: High returns penalized

Secondary: very bad returns not sufficiently penalized

Down-Side “Sharpe” Scores
(Sortino Measures)

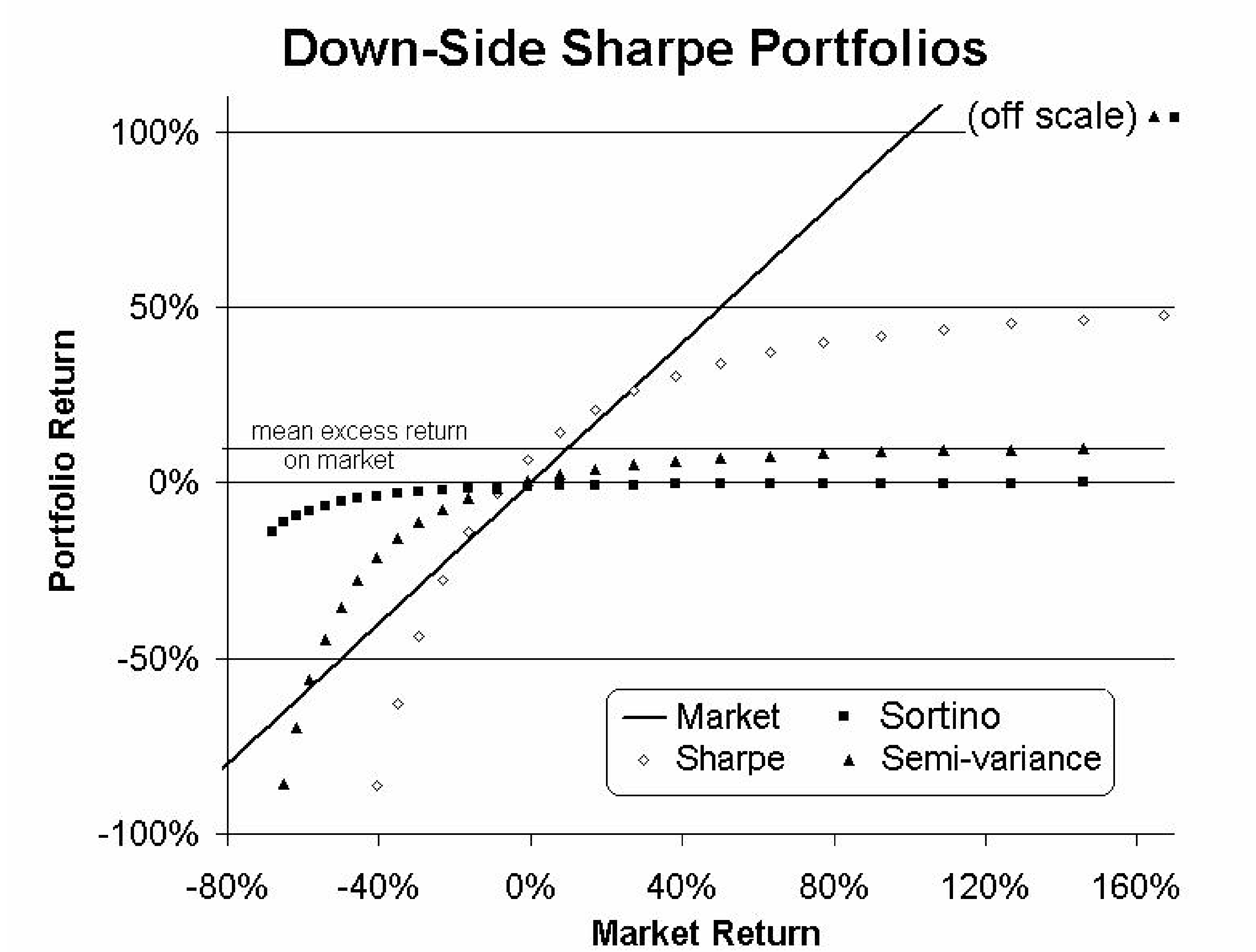
Risk measured by

  Semi-variance
or
  Root-Mean-Square down-side

Down-Side "Sharpe" Scores

$$S_{SV} = \frac{\bar{x}}{\left[\sum p_i [\mathrm{Min}(x_i - \bar{x}, 0)]^2\right]^{1/2}}$$

$$S_{Sor} = \frac{\bar{x}}{\left[\sum p_i [\mathrm{Min}(x_i, 0)]^2\right]^{1/2}}$$



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Up-Side Potential "Sharpe" Ratio

(van der Meer, Plantinga and Forsey)

$$S_{UP} = \frac{\sum p_i \cdot \text{Max}(x_i - x_{\text{MAR}}, 0)}{\left[\sum p_i [\text{Min}(x_i - x_{\text{MAR}}, 0)]^2\right]^{1/2}}$$

Portfolio is identical to maximal Sortino or semi-variance risk portfolio.

Benchmarked "Sharpe" Measures

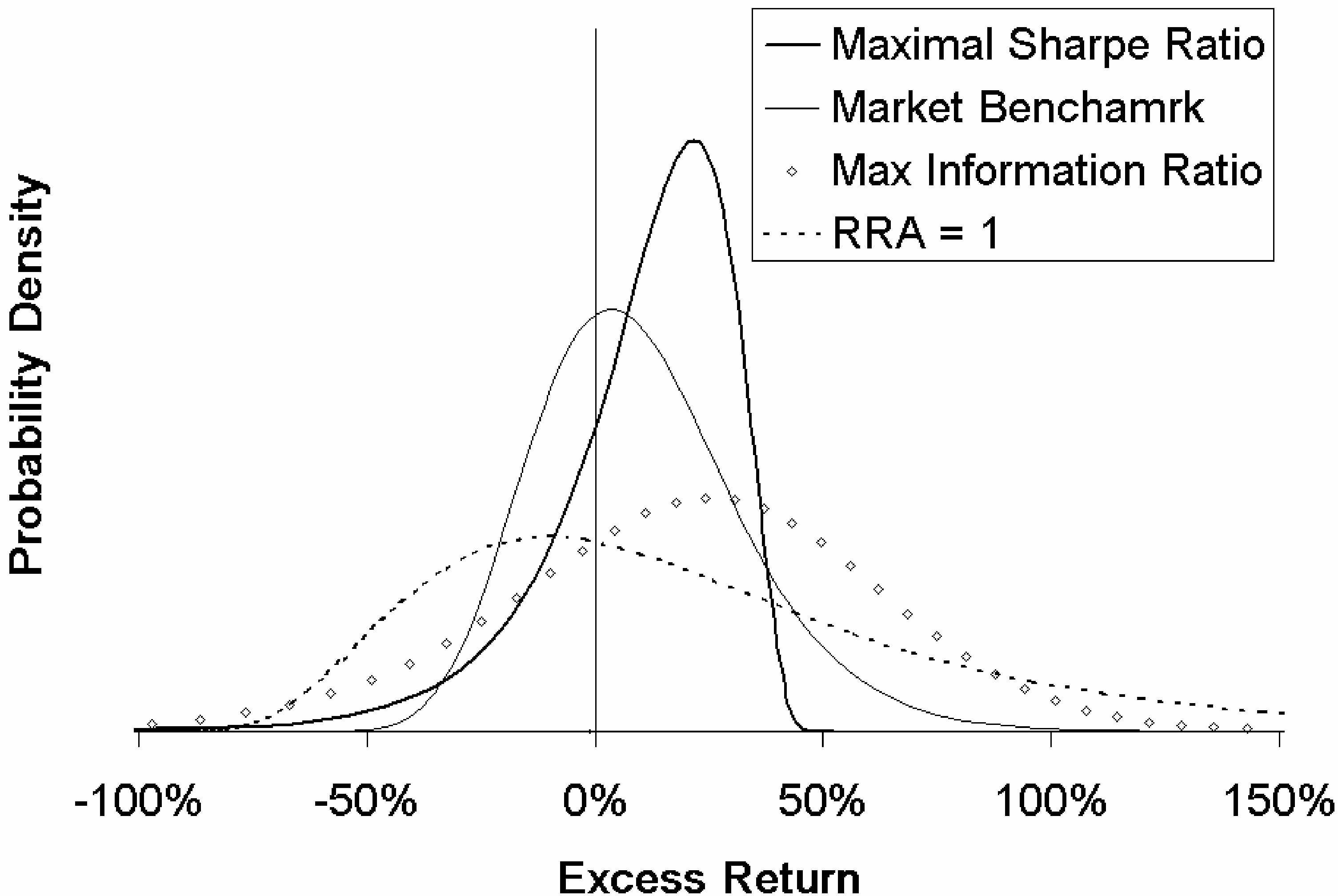
Sharpe's Information Ratio

$$S_{\mathrm{IR}} = \frac{\bar{x} - \bar{m}}{\left[\sum p_i (x_i - m_i)^2\right]^{1/2}}$$

m is the excess return on a benchmark like the market

🌢$_{\mathrm{IR}}$ is just the Sharpe ratio for $x - m$

Distributions of Portfolio Returns



Other Benchmarked Scores

The information ratio can be modified in the same way with down-side risk or upside potential

$$S_{\text{IR-Sor}} = \frac{\bar{x} - \bar{m}}{\left[\sum p_i [\mathrm{Min}(x_i - m_i, 0)]^2\right]^{1/2}}$$

$$S_{\text{IR-UP}} = \frac{\sum p_i \,\mathrm{Max}(x_i - m_i, 0)}{\left[\sum p_i [\mathrm{Min}(x_i - m_i, 0)]^2\right]^{1/2}}$$

Problem with Information Ratios

Impossible for a portfolio to be <u>measured</u> as less risky than the benchmark.

Same average return as benchmark with lower risk (variance) has a zero 🌢$_{IR}$

Lower average return as benchmark with much lower risk has a negative 🌢$_{IR}$

Sharpe Ratio with Information

Information improves portfolio through adjustments.

Purpose: To increase average return.

Side affect: Altering the portfolio increases the measured variance.

Informed Sharpe Ratio Example

Market return is normally distributed

Manager knows fraction η of uncertainty

Manager optimally adjusts portfolio levering it so holding or market is proportional to informed mean and inversely proportional to informed variance

Informed Sharpe Ratio Example

$$\text{Optimal holding} \propto \frac{\mu_{\text{inf}} - r_f}{\sigma_{\text{inf}}^2}$$

$$\text{measured Sharpe} = \frac{\mu_{\text{uninf}} - r_f}{\sigma_{\text{uninf}}}$$

Informed Sharpe Ratio Example

Measured Sharpe relative to market

$$\frac{S_i}{S_m} = \frac{1 + \eta / S_m^2}{\sqrt{1 + 3\eta + \eta(1+\eta) / S_m^2}} \text{ ¤ } 1$$

note: for fat-tails 3 is replaced by larger number

Informed Sharpe Ratio Example

$$\frac{S_i}{S_m} = \frac{1+\eta/S_m^2}{\sqrt{1+3\eta+\eta(1+\eta)/S_m^2}}$$

Example S_m = 0.75, the Sharpe ratio is

decreasing in η for η < 24.75%.

Informed manager with η < 88.4% scores lower than an uninformed manager

Informed Sharpe Ratio Example

Suppose manager holds less of market when expected return is higher?

Keeps measured mean close to constant

$$\frac{S_i}{S_m} \approx \frac{1}{1-\sqrt{\eta}} > 1$$

Informed Sharpe Ratio Example

$$\frac{S_i}{S_m} \approx \frac{1}{1-\sqrt{\eta}} > 1$$

Always exceeds 1

Larger for better information (higher η)

Suggestions

Show histogram of returns

not numerical

Suggestions

Report average increase in utility for various levels of risk aversion

$$\text{avg}\left[\frac{(1+r_f+x_i)^{1-\gamma}}{1-\gamma}\right]$$

difficult to explain and interpret.
not a single ranking

Suggestions

Report risk-adjusted returns

$$1 + r_f + \text{avg}[\tilde{x}] - \tfrac{1}{2}\gamma\sigma_x^2$$

Hard to explain

Suggestions

Report geometric mean return $\gamma = 1$

Report cumulative returns