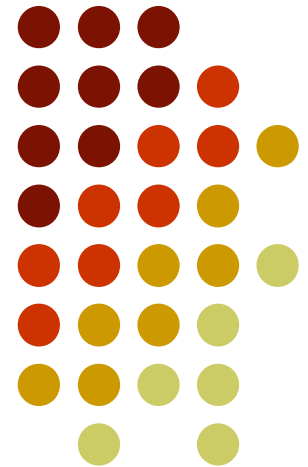


Valuing Correlation-Dependent Credit Derivatives

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IAFE Event, New York
May 2006





Road Map

- Review of Standard Market Model
- A New Implied Copula Approach
- Results from Using the New Approach



Standard Market Model

- Factor-based Gaussian copula model
- All companies have a hazard rate that is consistent with the index. This defines their time to default distribution
- Define

$$x_i = aM + \sqrt{1 - a^2} Z_i$$

where M and Z_i have standard normal distributions

- Map x_i on a quantile-to quantile to the the time to default for company i
- Copula correlation is a^2

Sample Data: August 30, 2005



CDX IG Tranches						
	0% to 3%	3% to 7%	7% to 10%	10% to 15%	15% to 30%	Index
5-year Quotes	40%	127	35.5	20.5	9.5	50

iTraxx Tranches						
	0% to 3%	3% to 6%	6% to 9%	9% to 12%	12% to 22%	Index
5-year Quotes	24%	81	26.5	15	9	36.375
10-year Quotes	53%	395	90	52	29	57.625

Implied Correlations for 5-year Quotes



	CDX IG					
	0% to 3%	3% to 7%	7% to 10%	10% to 15%	15% to 30%	Index
Tranche Correlation	0.091	0.012	0.068	0.106	0.156	n/a
Base Correlation	0.091	0.177	0.223	0.280	0.448	n/a

	iTraxx					
	0% to 3%	3% to 6%	6% to 9%	9% to 12%	12% to 22%	Index
Tranche Correlation	0.134	0.030	0.079	0.111	0.154	n/a
Base Correlation	0.134	0.210	0.266	0.308	0.428	n/a



Performance of Model

- If the model worked perfectly implied correlations would be the same for all tranche attachment points
- In fact implied tranche (compound) correlations exhibit a “correlation smile” and implied base correlations exhibit an upward sloping correlation skew.

Standard Market Model continued



- For valuing CDOs with non-standard attachment points we can make the Gaussian copula model work by interpolating between implied correlations
- However when valuing CDO squareds and other more exotic deals it is difficult to know the correct correlation to use.

Solving the Problem



- Find a copula that fits market prices better (e.g. the double t copula or copula where factor loading depend on factor values)
- Imply the copula from market prices

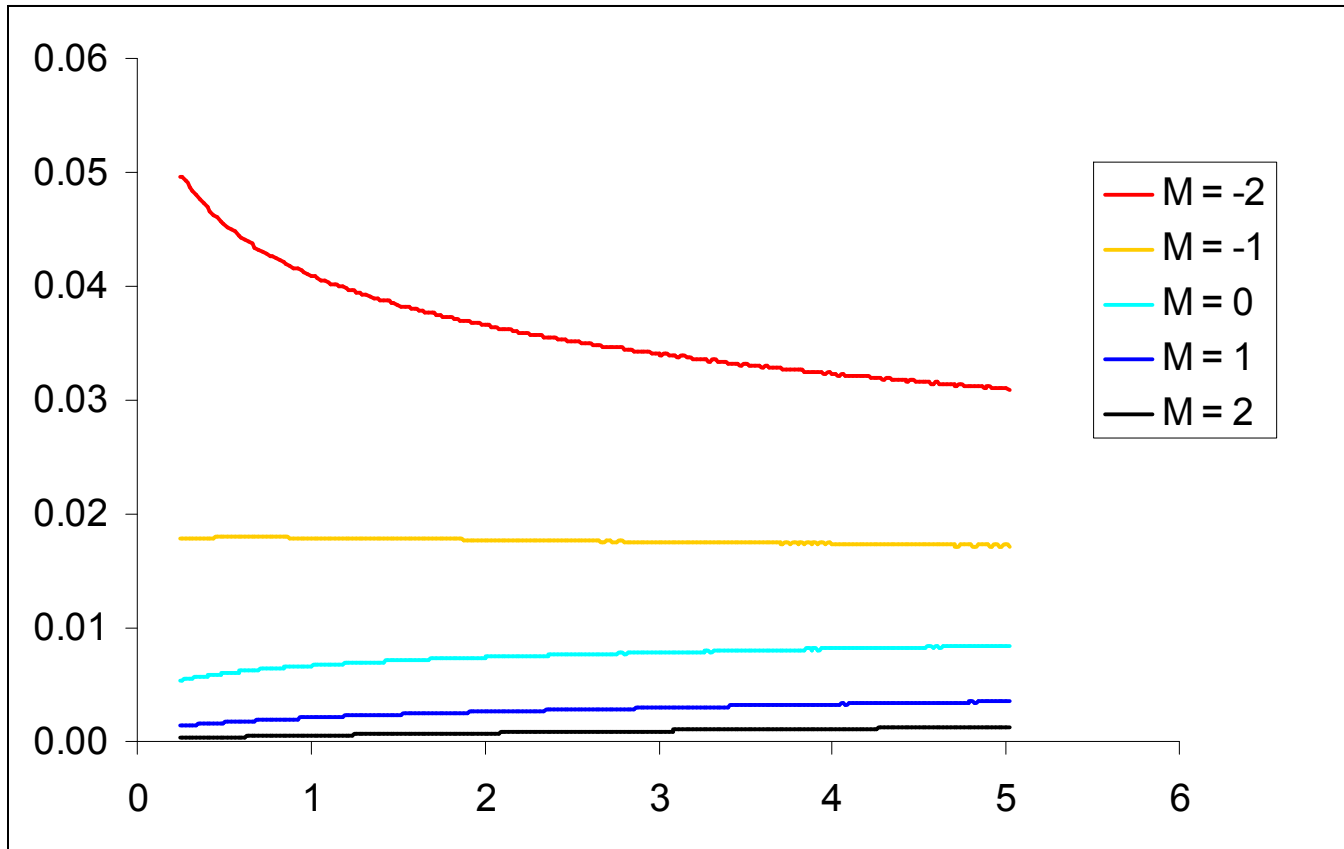
One way of Viewing Factor-Based Copula Model



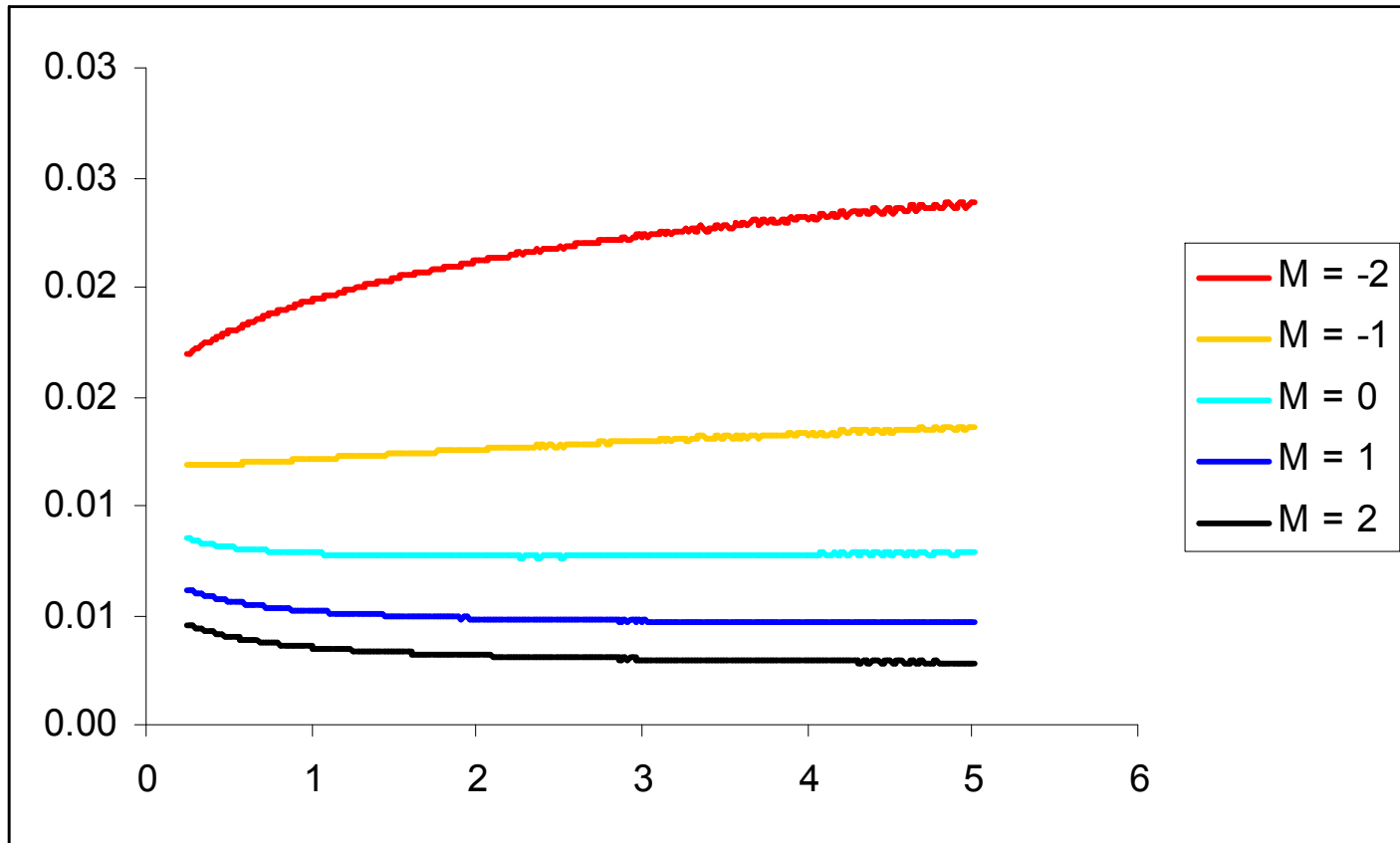
- We sample a value for the factor M
- The hazard rate as a function of time is then known for all obligors

Hazard Rate Paths for Gaussian Copula

(Unconditional hazard rate is 1% per year; implied correlation is 15%)



Hazard Rate Paths for Double t Copula (Unconditional hazard rate is 1% per year; implied correlation is 15%)



Implying the Copula



- Instead of specifying a factor-based copula we specify the hazard rate path probability distribution directly

The Steps for Calculating the Hazard Rate Distribution (Constant hazard rate case)



- Choose a set of values for the hazard rate (We specify a procedure for this) $\lambda_1, \lambda_2, \dots, \lambda_n$
- For each hazard rate path calculate PV of inflows and outflows for CDS and each CDO tranche
- Search for probabilities $\pi_1, \pi_2, \dots, \pi_n$ so that with market quotes the index and the CDO tranche prices are exactly zero. (Linear programming can be used to find the set of all such sets of probabilities.)

Optimization



We choose probabilities for the hazard rate paths so that the probability distribution of hazard rates is maximally smooth. We minimize

$$\sum_{i=2}^{n-1} \frac{(\pi_{i+1} + \pi_{i-1} - 2\pi_i)^2}{0.5(\lambda_{i+1} - \lambda_{i-1})}$$



Initial Assumptions

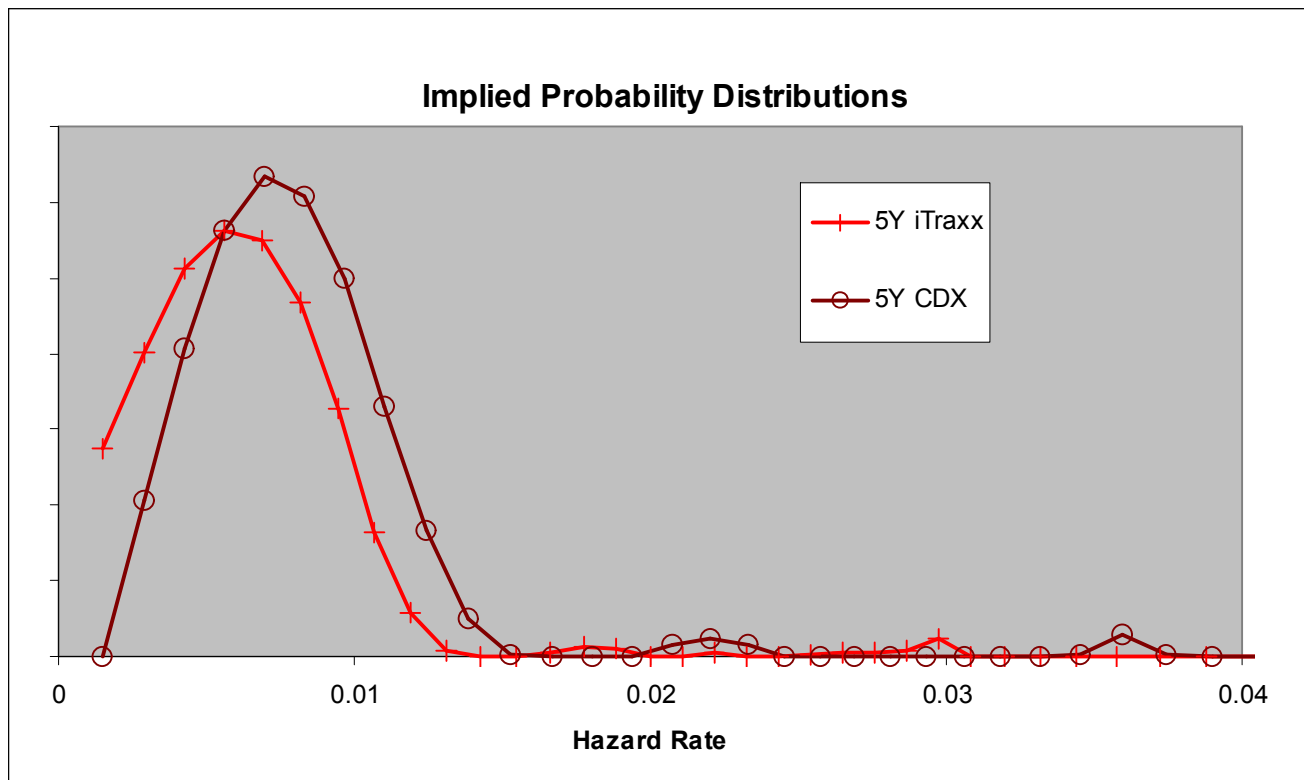
- Hazard rates are constant
- All companies have the same hazard rate
- Recovery rate equals 40%

No difficulty in fitting iTraxx and CDX data in 2004

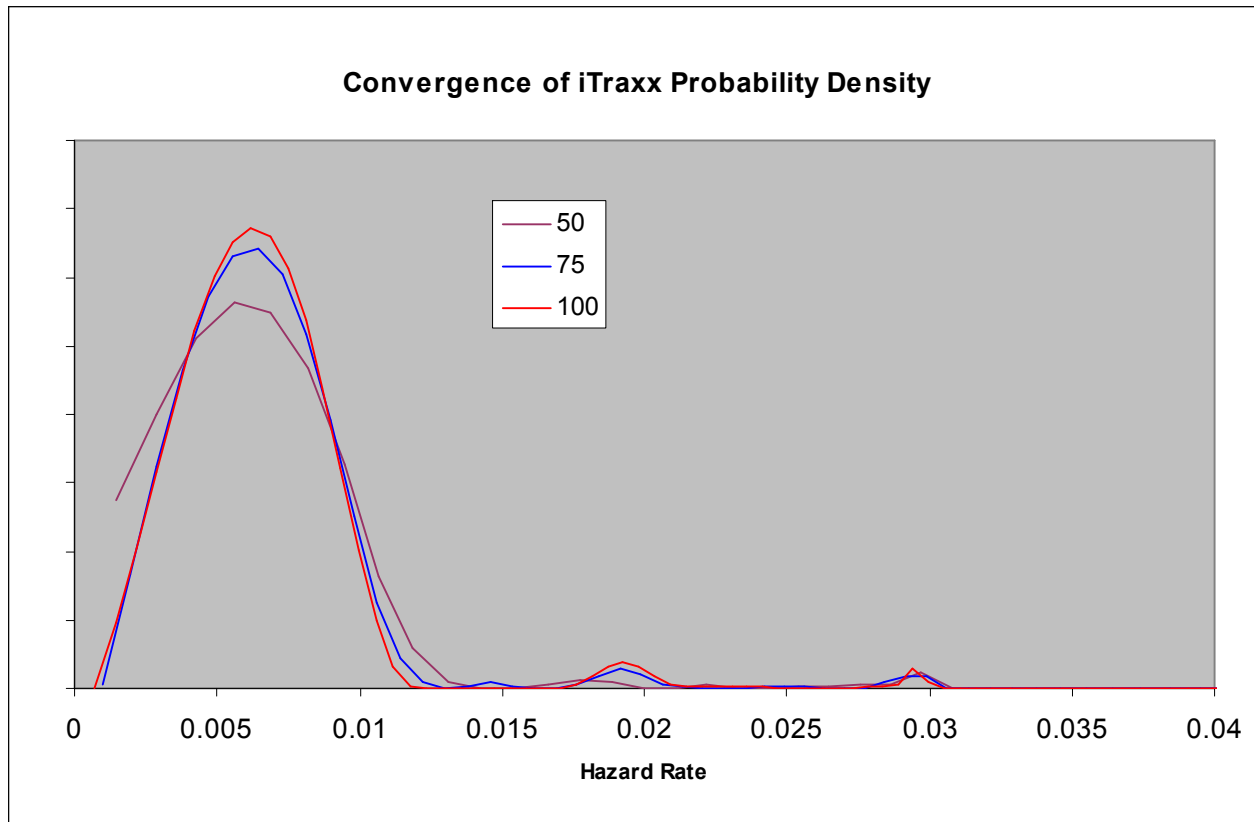
For more recent data we need to incorporate the best fit relationship between recovery rate and default rate documented by Moody's:

Recovery Rate = $0.52 - 6.9 \times$ One-year default rate

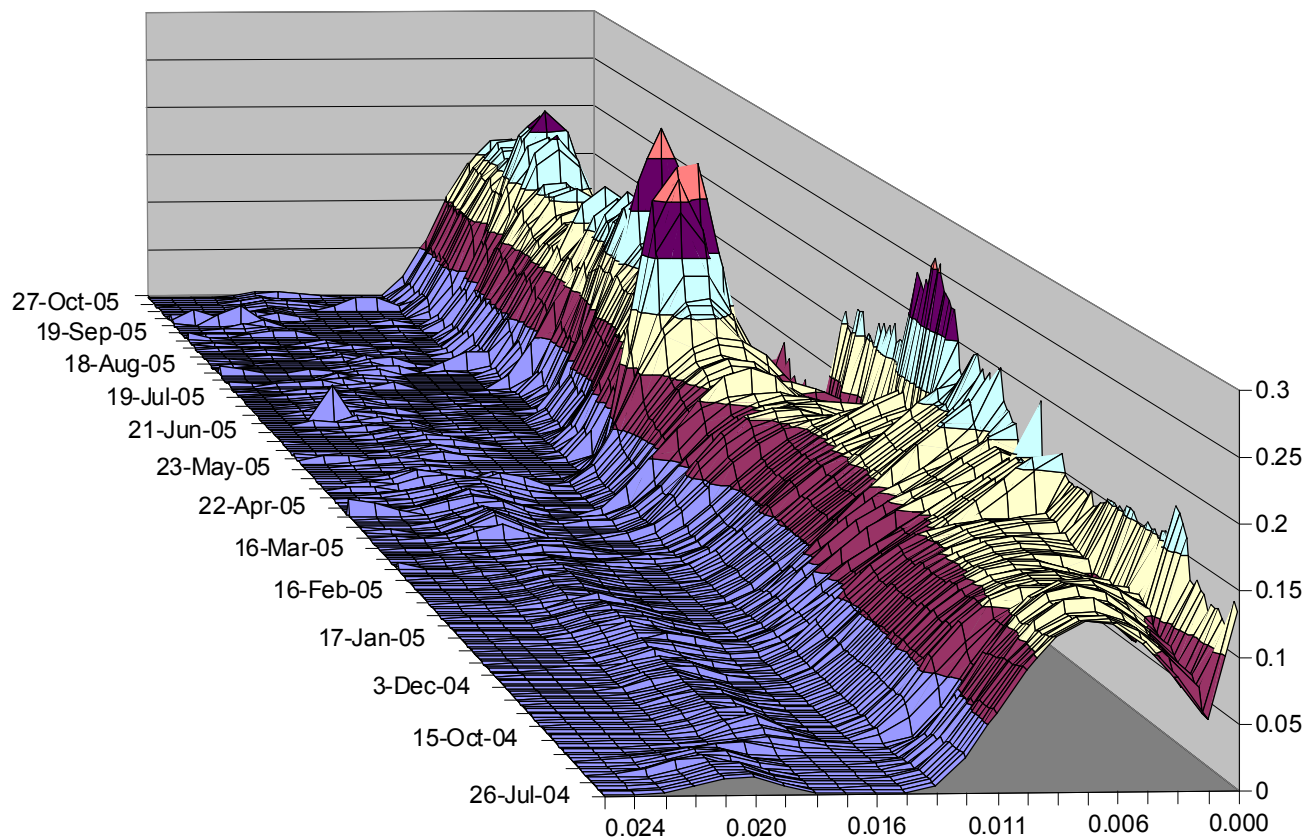
Probability Densities of Hazard Rate for iTraxx and CDX Hazard Rates Using 5-Year Data



Probability Density for iTraxx 5-year Hazard Rate as Number of Points is Increased



Hazard Rate Distribution for iTraxx, July 26, 2004 and November 2, 2005



Where have the Correlations Gone? What About Default Time Distributions?



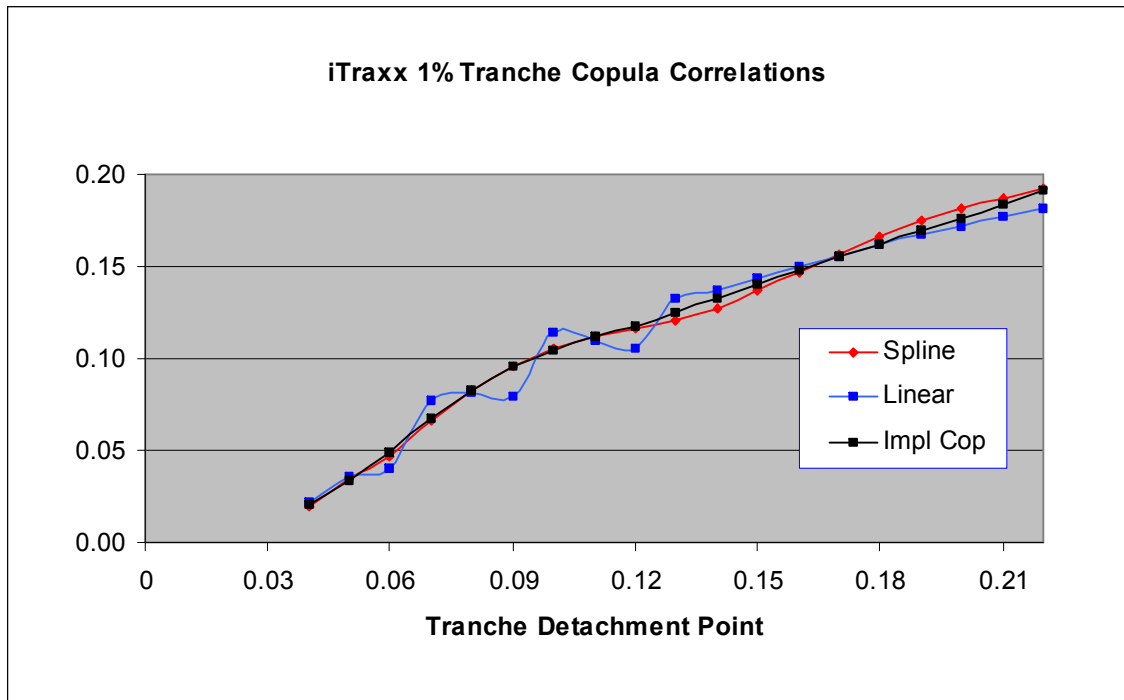
- The correlation depends on the dispersion of the hazard rate paths
- The default time distribution is defined by the shape of the hazard rate paths



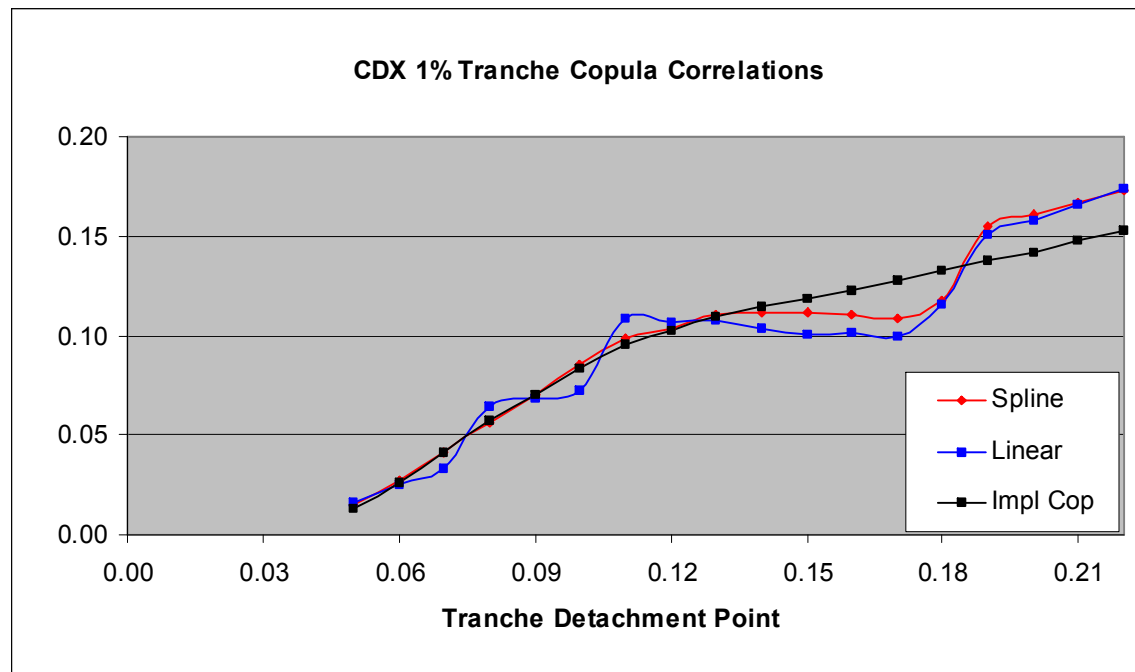
Uses of the Model

- Non-standard attachment/detachment points
- Non-standard life
- Non-standard portfolio
- Non-standard structure (e.g. CDO squared)

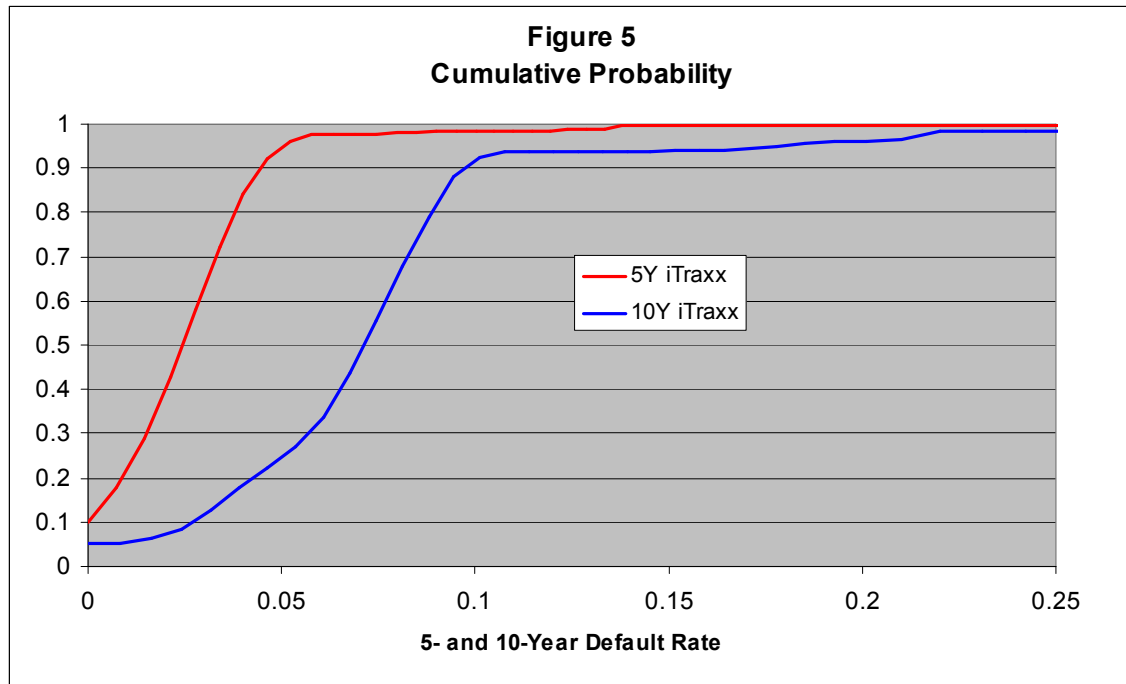
Implied Correlations for Base Correlation and Implied Copula Models For tranches of iTraxx that are 1% wide, Aug 30, 2005



Implied Correlations for Base Correlation and Implied Copula Models For tranches of CDX that are 1% wide, Aug 30, 2005



Cumulative Probability Distributions for 5- and 10-year Default Rates for iTraxx



Effect on Spread of Varying the number of names in iTraxx



	Number of Firms in Portfolio						
	40	60	80	100	125	150	200
Perfect Copula	154.4	122.4	103.7	91.6	81.0	75.2	67.0
Normal Copula: Tranche Correlation	174.3	133.3	109.4	94.0	81.0	73.1	62.8
Normal Copula: Base Correlation	124.8	103.9	93.4	86.8	81.0	77.6	73.0

Spreads for CDO Squareds (10 child portfolios, 80 names each, overlap= 23% on average)



Child Tranches	Parent Tranche										CDO
	0-10%	10-20%	20-30%	30-40%	40-50%	50-60%	60-70%	70-80%	80-90%	90-100%	
3-6%	499	190	104	68	52	44	39	35	31	24	104
6-9%	53	37	33	30	28	25	22	19	15	12	27
9-12%	29	24	20	17	14	13	11	10	10	9	16
Mixed 1	234	63	40	33	29	25	20	16	12	10	
Mixed 2	3063	1056	395	140	65	43	35	30	24	16	

Implied Correlations for CDO Squareds (10 child portfolios, 80 names each, overlap= 23% on average)



Child Tranches	Parent Tranche										CDO
	0-10%	10-20%	20-30%	30-40%	40-50%	50-60%	60-70%	70-80%	80-90%	90-100%	
3-6%	0.24	0.019	0.024	0.028	0.035	0.043	0.052	0.062	0.076	0.094	0.026
6-9%	0.036	0.056	0.068	0.079	0.088	0.095	0.101	0.106	0.112	0.122	0.071
9-12%	0.079	0.098	0.105	0.108	0.11	0.112	0.117	0.124	0.134	0.151	0.106
Mixed1	0.012	0.033	0.05	0.068	0.083	0.096	0.104	0.11	0.116	0.132	
Mixed 2	0.112	0.117	0	0.021	0.053	0.06	0.067	0.078	0.094	0.109	

Effect of Overlap on CDO Squared Spreads



Breakeven tranche rate for CDO-squared tranches with different degrees of overlap.					
			Overlap		
Tranche	AP	DP	23%	46%	70%
1	0%	10%	498.7	428.3	359
2	10%	20%	190.2	187.6	183.7
3	20%	30%	103.6	116.1	127.1
4	30%	40%	68.3	81.4	94.9
5	40%	50%	52.3	62.5	74.3
6	50%	60%	43.9	51	60.3
7	60%	70%	38.7	43.6	50.1
8	70%	80%	34.6	37.8	43
9	80%	90%	30.5	32.9	37.1
10	90%	100%	23.7	26.5	30.3



Summary

The implied copula model has a number of advantages:

- Exact fit to market quotes
- More intuitive than Gaussian copula models
- Can calculate Greeks with respect to CDS and CDO tranche quotes
- Can be used for non-standard deals
- Provides a tool for a trader to reflect market beliefs



New Book!

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