# A New Framework for Analyzing and Managing Macrofinancial Risks of an Economy

By

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# Abstract

The high cost of international economic and financial crises highlights the need for a comprehensive framework to assess the robustness of national economic and financial systems. This paper proposes a new comprehensive approach to measure, analyze, and manage macroeconomic risk based on the theory and practice of modern contingent claims analysis (CCA). We illustrate how to use the CCA approach to model and measure sectoral and national risk exposures, and analyze policies to offset their potentially harmful effects. This new framework provides economic balance sheets for inter-linked sectors and a risk accounting framework for an economy. It also provides a new framework for sovereign capital structure analysis. It is useful for assessing vulnerability, policy analysis, risk management, investment analysis, and design of risk control strategies. Both public and private sector participants can benefit from pursuing ways to facilitate more efficient macro risk accounting, improve price and volatility discovery, and expand international risk intermediation activities.

# A New Framework for Analyzing and Managing Macrofinancial Risks of an Economy

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## Introduction

The vulnerability of a country's economy to volatility in the global markets for credit, currencies, commodities, and other assets has become a central concern for economic policymakers everywhere. The high cost of international economic and financial crises highlights the need for a comprehensive framework to assess the robustness of national economic and financial systems. This paper proposes a new approach to measure, analyze, and manage macroeconomic risk (credit, market, and other risks) in a comprehensive way based on the theory and practice of modern contingent claims analysis (CCA).

In the proposed analytical framework, we view the sectors of a national economy as interconnected portfolios of assets, liabilities, and contingent claims. We measure the values of sector portfolios and the sensitivities of the market values of these sector portfolios to changes in underlying market risk factors and risk transmission between sectors. We illustrate how to use contingent claims analysis to model and measure sectoral and national risk exposures, and analyze policies to offset their potentially harmful effects.

## Contingent Claims Analysis

A contingent claim is any asset whose future payoff is contingent on the values of other assets. The essence of the CCA approach is to analyze how the value of the contingent claim changes as the values of the underlying variables change continuously through time. Modern contingent claims analysis is a generalization of option pricing theory.[1] In the seminal articles on option pricing, Black-Scholes (1973) and Merton (1973) showed that the total value of a firm is equal to the value of the securities in the capital structure which can be viewed as implicit options. Since 1973, the Black-

[1] See Merton (1992, 1998).

Scholes-Merton methodology has been applied to a wide variety of contingent claims. Levered equity can be modeled as a call option on assets and risky debt can be modeled as default-free value of debt less a put option on assets.[2] The debt of a firm is a claim on the value of the underlying assets of the firm. The total market value of the firm's asset is equal to the market value of equity plus the market value of debt. If the asset value declines below the point where debt payments on scheduled dates cannot be made, default is the result. The default-free value of debt represents the default barrier. Holders of equity have a contingent claim on the residual value of the firm's assets in the future, which can be modeled as an implicit call option on the assets of the firm.

Financial guarantees can be analyzed as put options.[3] A debt claim of any type – a loan, a bond, etc. — satisfies the following equation:

Risky debt + guarantee against default = Risk-free debt

Or equivalently,

Risky debt = Risk-free debt - guarantee against default

This is an identity that holds both conceptually and in terms of value. If the debt is collateralized by a specific asset, then the guarantee against default can be modeled as a put option on the asset with an exercise price equal to the face value of the debt. The debt holder is offering an implicit guarantee as it is obligated to absorb the losses if there is default. However, often a third party is the guarantor, as is the case when government guarantees the deposit liabilities of banks or the pension-benefit promises of firms.

By applying this analytical framework to the assets and liabilities of entities at different levels of aggregation – industries, sectors, and whole national or regional economies – it is possible to analyze the dynamics of value changes in response to changes in underlying economic variables.

[2] See Merton 1974.
[3] See Merton, 1977 and Bodie & Merton, 2000.

## General Framework for Contingent Claims Balance Sheets

The liabilities of a firm, a portfolio of firms in a sector or the public sector (combined government and monetary authorities) can be valued as contingent claims on the assets of the respective firm, sector or public sector. The contingent claims approach is based on three principles: (i) the values of liabilities are derived from assets; (ii) liabilities have different priority (i.e. senior and junior claims); and, (iii) assets follow a stochastic process. Contingent claim balance sheets are balance sheets based on the contingent claim relationships where assets are always equal to the (market) value of liabilities. The liabilities consist of senior claims (such as senior debt), subordinated claims (such as subordinated debt) and the junior claims (equity or the most junior claim). For firms and financial institutions, equity is the junior claim. However, the more general term "junior claim" will be used to describe the most junior claim for the public sector and equity of firms and banks. This more general description will help clarify analysis when we apply the framework to the public sector or other sectors such as households.

The value of liabilities is derived from assets in the CCA framework so the changes in the value of senior and junior claims are derived from stochastic changes in the asset value. Random variations in asset prices over time require the use of specific financial mathematical techniques to model random processes, as opposed to the deterministic processes. Total assets, being stochastic, have a chance of declining to or below the level of the default-free value of debt (i.e. the default barrier). As total assets decline relative to the default barrier, the implicit put option (i.e. implicit guarantee) rises and the value of risky debt declines. Simultaneously, the value of the junior claim (call option) declines. Credit spreads on risky debt are a function of the assets, default barrier, volatility of assets, time and risk-free interest rate. (See Annex 1 for more details.)

## Contingent Claim Balance Sheets for Sectors

We view an economy as a set of interrelated balance sheets with three types of aggregate sectors -- corporate, financial, and public sector.[4] The corporate sector refers to an aggregation of all non-financial firms.

### Corporate Sector and Sub-Sector CCA Balance Sheets

The same general principles of contingent claims that apply to analysis of a single firm can also be applied to an aggregation of firms in sector or sub-sectors. The sector or sub-sector equity value can be valued as a call option on the sector or sub-sector assets with the default barrier derived from the default-free value of corporate debt which includes domestic and foreign debt. The market value of debt is the default-free value of debt minus a put option.

### Financial Sector and Sub-Sector CCA Balance Sheets

Similarly, we can apply the CCA framework to financial sub-sectors, such as banks, pension funds and insurance companies. Equity of the sub-sector is a contingent claim on total assets, which is modeled as a call option. Liabilities include debt and deposits (for banks). The financial guarantee from the government is a contingent asset, which is modeled as a put option. Governments and central banks typically provide explicit or implicit financial support to large financial institutions in the case of serious deposit runs, illiquidity or insolvency.

### Public Sector CCA Balance Sheet

The public sector balance sheet is an analytical economic balance sheet of the combined government and the monetary authorities.[5] To construct a CCA balance sheet of the public sector, we need to identify the eligible assets and liabilities, and establish their interrelationship and priority. Within this framework, the securities issued by the

[4] Gray, Merton, Bodie (2002); Draghi, Merton, Giavazzi, (2002); Gray (2002).

[5] This analytical combined balance sheet includes the monetary authority activities related to foreign currency reserves and "net domestic credit" to government but excludes the direct activities of the monetary authority with the banking sector, such as credit and liquidity support activities that do not go through the government balance sheet or affect foreign exchange reserves.

public sector and the financial guarantees provided by it give the holders contingent claims on the assets of the government and monetary authorities. The goal is to construct the liability side of the balance sheet so that the liabilities can be valued and linked to the value of total assets.

A useful starting point is to list the main entries on the combined balance sheet of the government and monetary authorities.[6]

Assets include:

- Foreign currency reserves and contingent foreign currency reserves;[7]
- Present value of taxes and revenues;
- Other public assets (equity in public enterprises, land, mineral assets, and social overhead capital);
- Value of the public sector's monopoly on the issue of money.

Liabilities include:

- Present value of government expenditures (including social insurance and other entitlement programs);
- Local-currency debt;
- Foreign-currency debt;
- Financial guarantees;
- Base money.

It is useful to look into the relationships between items in four categories: fiscal activities, monetary and foreign currency reserve activities, risky debt, and financial guarantees.

*Fiscal Assets and Liabilities* - In this framework, the items related to fiscal assets and liabilities are taxes, revenues and expenditures. Expenditures can be divided into non-discretionary expenditures which are senior claims, and discretionary expenditures which are junior claims. Non-discretionary expenditures are core expenditures (e.g., defense, education, core infrastructure, etc.) that will not be given up before giving up on paying the debt. Operationally, discretionary expenditures are ones that are subordinated to the explicit liability claims against the government.

[6] Buiter (1993) includes most all of these items on the accounting balance sheet of the public sector.
[7] The total foreign reserves of the public sector include actual reserves plus *contingent* reserves from international financial institutions, such as the IMF, or contingent credit lines.

Discretionary expenditures may become especially significant in situations of high fiscal revenue such as windfalls from oil or natural resources.

Under stress situations, the government maintains the non-discretionary expenditures and cuts the discretionary expenditures. Under these assumptions, we can subtract the present value of non-discretionary expenditures from the present value of tax capability to obtain the *net fiscal asset*,[8] given that non-discretionary expenditures are clearly senior claims and have the same maturity patterns as taxes and fiscal revenues. The *net fiscal asset* is thus similar to the present value of the primary fiscal surplus over time (the present value of fiscal surplus minus interest payments). This step also simplifies the process of constructing the CCA balance sheet because it is much easier to obtain market values for the other non-expenditure related liabilities, as will be discussed in more detail later.

*Monetary and Foreign Reserve Assets and Liabilities* - We derive the value of money on the asset side of the public sector balance sheet from the value of the public sector monopoly on issuing money. This is equivalent to the present value of seigniorage. Base money is a liability of the public sector.[9] The holders of money derive liquidity and convenience yield from money. (See Annex 2 for more details on the value of money on the public sector balance sheet.)

In the framework described in this paper, however, the authors note that there is an additional component to the value of money related to the option value of exchanging money for foreign currency. The exchange of money for foreign currency results in a drop in the net foreign assets of the banking system, leading to a direct change in the foreign currency reserves of the monetary authorities.[10] This option component of the value of money can be valued as a call option on foreign reserves (including contingent foreign reserves) available after higher priority claims on such reserves. This exchange is

[8] The value of assets of an operating firm can be considered as the present value of stochastic future cash flow from income minus net new investment expenditures to create that income. For the public sector, the net fiscal asset is the present value of stochastic future fiscal flows from taxes and revenues minus non-discretionary expenditures.

[9] Base money is also known as high-powered money or reserve money. As is the common practice, it is the main liability of the monetary authorities (IMF, 2000, Buiter, 1993, Blejer and Schumacher, 2000). Base money is "multiplied" by the banking system; the multipliers relate base money to M1, M2, etc.

[10] This is the standard operating assumption in the IMF financial programming, (IMF, 2000), and described by Schaechter, 2001.

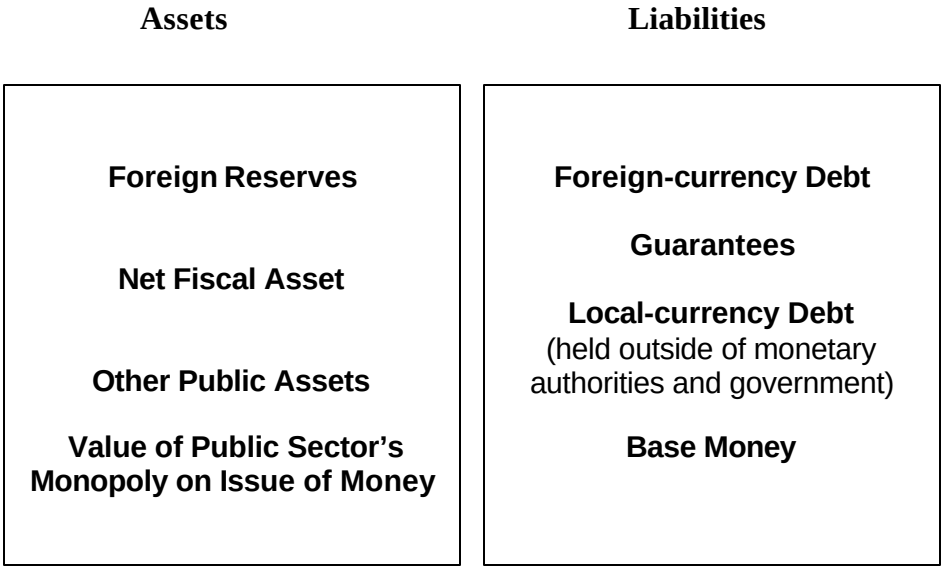
conditional on the willingness of the government sector to allow convertibility into foreign currency (see more detailed discussion in Annex 2.)

*Risky Debt Liabilities* - Risky debt includes foreign-currency debt and local-currency debt of the government. On the combined balance sheet of the public sector, the local-currency debt is the portion of debt held outside of the monetary authorities and the government.

*Financial Guarantees* – As described earlier, implicit or explicit guarantees to "too-important-to-fail" banks and other financial institutions or pension obligations are liability items on the public sector's balance sheet which are modeled as put options.

The combined balance sheet for the public sector is shown in Figure 1. The numeraire can be in local or foreign currency units. The CCA balance sheets for large developed economies with "hard" currencies are measured in units of local currency. The CCA balance sheets of emerging market countries with "soft" currencies are usually measured in a "hard" currency (e.g. US dollar) because it simplifies the analysis and we are interested in valuation and credit risk associated with claims denominated in hard currencies, such as foreign-currency denominated debt.

## Figure 1 – Combined Balance Sheet for the Public Sector



The public sector balance sheet shown above is derived from separate balance sheets of the government and the monetary authorities where there are cross-holdings and financial guarantees between these two public sector "partners".[11] Under this structure, the assets of the monetary authority include foreign reserves, claims on government local-currency debt and other obligations. The liabilities of the monetary authority partner are base money[12] and financial guarantees to the government, including guarantees to supply foreign currency to service the sovereign foreign-currency denominated debt. The assets of the government partner include the net fiscal asset, the present value of seigniorage, and other assets, while the liabilities include local-currency debt held by the monetary authority, local-currency debt held outside of the monetary authority, financial guarantees and foreign debt. When the separate balance sheets of the two partners are combined, the end result is the combined public sector balance sheet shown in Figure 1. The local-currency debt and the base money are modeled as junior claims and foreign-currency debt and guarantees as senior claims. (See Annex 3 for more detailed discussion).

The most senior liabilities of the public sector are foreign-currency debt and guarantees in Figure 1. Foreign-currency debt is shown as senior debt because many governments appear to make strenuous efforts to meet such payments. Emerging market borrowers with market access usually (but not always) attach a high priority to service foreign-currency debt obligations. Local-currency debt and the guarantees of currency convertibility tend to be junior to the foreign-currency debt obligations. For example, as the value of public sector assets declines, the government tends to impose restrictions on currency convertibility and restructure its local-currency debt to maintain payments on foreign-currency debt.[13] Convertibility protection prices are frequently fifty to one

[11] This balance sheet can be framed along the lines of a corporate parent-subsidiary structure. The monetary authority partner might be viewed as similar to the "parent" with cross-holding and guarantees to the government which is the "subsidiary." In any case, this is the same as "partners" with cross-holdings and guarantees described in this paper.

[12] Base money is issued with the associated obligation of the monetary authorities to exchange base money for foreign currency reserves, in the absence of capital controls. When holders of money exchange it for foreign currency, this leads to a reduction in the foreign reserves of the banking system which translates into a reduction in the foreign reserves of the monetary authorities.

[13] Personal communications with sovereign credit analysts from Moody's Investors Service, the IMF and World Bank. Argentina's recent default was preceded by convertibility restrictions and "forced" restructuring of local-currency debt held by banks. Ecuador had a similar sequence (Beckerman, 2002).

hundred percent higher than credit default protection prices and analysts "feel the likelihood of a convertibility default is greater than default of a sovereign on its debt obligations."[14] However, if the asset value continues to decline, this may lead to default on foreign-currency debt. The priority of debt service obligations can, however, vary among countries and over time within a country.[15]

In this framework, the public sector's liabilities are contingent claims modeled with implicit options. The local-currency debt of the government, held outside of the monetary authority, and the base money can be modeled as a call option on total public sector assets with the default barrier being senior liabilities – foreign currency debt and guarantees entities. (See Annex 2 and 3 for further details.)

Interlinked CCA balance sheets for the corporate sector, the financial sector and the public sector are shown in Figure 2.[16] For emerging markets, this is a very useful framework to analyze the value of the public sector's liabilities, especially foreign debt, as well as risk transmission between sectors. For developed market economies, where foreign debt valuation is not a major focus, this framework is useful for the analysis of the level and the rate of change of the guarantees between the public and the financial sectors, and the risk transmission between sectors. Moody's-MfRisk is the first commercial application of this CCA framework which has been applied to a number of developing and developed countries.

When the government takes steps to strengthen its fiscal position (e.g., increase the fiscal asset and refrain from printing money), this increases the value of the local-currency and foreign-currency debt since their value is derived from assets.[17] This framework facilitates a comprehensive analysis and risk management of all components of the public sector balance sheet.

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[14] Tavakoli, 2001, quotes for Mexico, Brazil and Argentina and observations by H. Chan, p 212-220.
[15] For political reasons, a new government might reorder the priority. This framework can, in principle, be adapted to handle different priorities.
[16] The household sector balance sheet can be added, with household income and assets comprising assets. Household non-discretionary expenditures are the senior liability, debt as a subordinated obligation and discretionary expenditures of households being the junior claim.
[17] In developed market economies, the size of the local bond market is larger and maturities are longer than in emerging markets. Developed countries are likely to be able to issue debt denominated in their own "hard" local currencies in foreign markets, while this is very difficult for "soft" currencies like those of emerging markets. These risk impact of these differences can be analyzed with this CCA framework.

## Figure 2 – Interlinked CCA Balance Sheets for the Economy
**(Measured in US $)**

| Assets | Liabilities |
|---|---|
| **CORPORATE SECTOR** | |
| **Corporate Assets** | **Debt** (default-free value minus put option)<br>**Equity** (call option on corporate assets) |
| **FINANCIAL SECTOR** | |
| **Loans and other Assets** (including loans to corporate sector and public sector)<br>**Financial Guarantees** (modeled as put option) | **Debt / Deposits / Liabilities** (default-free value minus put option)<br>**Equity** (call option on financial sector total assets) |
| **PUBLIC SECTOR (Combined Government and Monetary Authorities)** | |
| **TOTAL ASSETS**<br>**Foreign Currency** (including contingent foreign reserves)<br>**Net Fiscal Asset and Other Public Assets**<br>**Value of Public Sector's Monopoly on Issue of Money** | **Foreign-currency Debt** (default-free value of debt minus put option)<br>**Financial Guarantees** (modeled as put options related to too-important-to-fail financial and other entities)<br>**Base Money and Local-currency Debt** Held Outside of the Government and Monetary Authorities (call options on public sector assets) |

## Measuring Implied Asset Value and Volatilities Using Market Prices

There are a number of ways to measure asset value and volatility such as market observed prices, comparables, and implied values from market prices. By using current market information of security prices to calculate the implied asset values, the CCA approach overcomes some of the drawbacks of relying exclusively on past pricing and accounting data. From the observed prices and volatilities of market-traded securities, one can estimate the implied values and volatilities of the underlying assets.[18] These implied asset values and asset volatilities can be used to calibrate the pricing and risk model of major sectors in the economy.

Domestic equity markets provide pricing and volatility information for the calculation of corporate and bank assets, and asset volatilities. The prices in the international markets (including foreign currency market, debt market, and credit derivatives market), together with information from domestic market prices, provide the market information for the value and volatility of liabilities on the public sector balance sheet.[19] We then use this information to calculate implied asset values, volatilities, and higher moments of implied asset distributions (see Annex 4), and calibrate the economic CCA balance sheets. Subsequently, we can do "forward" simulations to estimate the impact of "shocks" and policy changes on the economic and financial system.[20]

## Measuring Risk Exposures

So far, we have discussed how to calculate the value of debt, guarantees, and equity using the CCA approach. We now turn to how to measure the risk exposures. Given that income or cash flow accounts and mark-to-market balance sheets are designed to measure cash flows and value, they do not give useful information about the risk exposures, or the sensitivity of value changes or losses in the future.

However, the values of the contingent claims on the CCA balance sheets contain embedded implicit options which can be used to obtain certain risk measures. These include the sensitivity of the implicit option to the underlying asset (the delta),

[18] See Bodie and Merton (1995).
[19] Gray, 2000, 2001.
[20] As in the Moodys MFRisk model

sensitivity to other parameters, distance to distress, spreads on debt, probabilities of default, risk exposures in risky debt, value-at-risk and other measures.

**Box 1**

## Measuring Risk Exposure and Vulnerability

There are a number of measures of risk and vulnerability including:

Option Sensitivity to the Underlying Asset (Delta): The delta is the change in the value of the implicit option with a change in the value of the underlying asset. The value of delta measures the exposure to the option (e.g. the government's exposure to the value of its guarantee as banking assets change).

Other Option Sensitivities: Other option sensitivity parameters ("Greeks") are useful risk measures. The "gamma" of an option is the change in the delta for a change in the underlying asset. Delta is the "slope" of the graph of option value vs. asset value, while the gamma is the "convexity." The "vega" of the option is also very useful as it gives the sensitivity of the option to the change in the volatility of the underlying asset.

Spreads on Debt: Credit spreads on debt can be calculated using formulas derived from the option equations. This spread is a function of the leverage ratio, volatility of assets, time and risk-free interest rate. The leverage ratio is the ratio of the asset to the present value of the default barrier, i.e. the default-free debt value.

Probabilities of Default: Probabilities of default can be calculated from the option formulas. These are useful for valuation of risky debt, credit default swaps and derivatives.

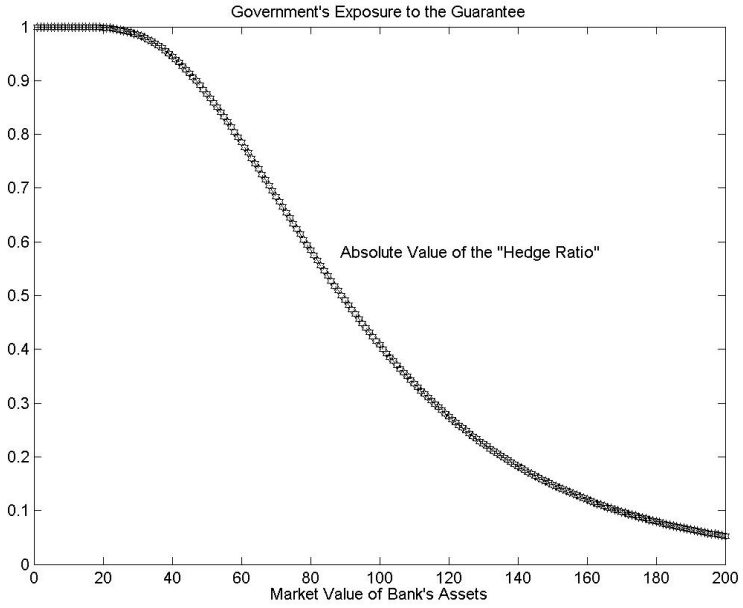
Measuring Risk Exposures in Risky Debt: Even if the loans are appropriately marked to market, there is an issue on how to measure the future risk exposure. The amount of money that one can reasonably expect to lose as a result of a default over a given period is normally called the "expected risk exposure." The probable loss will depend on: (i) the amount exposed to credit risk; (ii) the probability of counterparty defaulting; and, (iii) the recovery rate.

Value-at-Risk and Other Indicators: Value-at-Risk measures the maximum amount likely to be lost over a specific time period for a given confidence level. A variety of other risk indicators include sensitivity of distance to distress and implicit put and call options to changes in underlying parameters (e.g., exchange rates, interest rates, assets, volatility, time, etc.).

(See Annex 1 for additional details.)

The delta measures the change in the value of an option per a unit change in the value of the underlying asset. Option deltas are thus the appropriate measure of an entity's exposure to specific sources of risk. For example, the government's exposure to its guarantee to the banking sector changes as banking assets change. Figure 3 shows the absolute value of the delta for the guarantee plotted against the banking system asset. This is simply (the absolute value of) the slope of the tangent to the function defining the value of the option at any point. The delta measures the government's exposure. Figure 3 shows that the government's exposure, acquired through the guarantee, increases as the market value of the bank's assets falls.

**Figure 3: Government's Exposure to the Guarantee Measured by the Delta (or Hedge Ratio)**



## A Simple Three-Sector Framework

To show how we can apply the CCA framework, we focus on a simplified model with three aggregate sectors -- corporate, financial, and public sectors. In this model, the corporate sector's liabilities include bank loans which are the banking sector's assets. The liabilities of the banks are deposits that are insured against a sovereign default. The system's financial stability depends on the government's financial guarantee to the banks.

The inter-linkage between the sectors is straightforward. The debt of the corporate sector can be described as default-free debt combined with a short of a put option on corporate assets. The economic balance sheet of the banking sector has assets consisting of corporate loans (default-free debt minus the value of a put option). The banking sector also includes guarantees from the government as an asset, which is a liability on the government's economic balance sheet. This is shown in Figure 4.

## Figure 4 Balance Sheets for Simple Three-Sector Framework

**Corporate Sector Balance Sheet**

| Assets | Liabilities |
|---|---|
| Corporate Assets | Debt (=Default-free value of debt minus implicit put option) |
| | Equity (Implicit call option) |

**Banking Sector Balance Sheet**

| Assets | Liabilities |
|---|---|
| Loans (Debt of Corporate Sector) | Debt |
| Other Assets | Deposits |
| Financial Guarantee (Implicit Put Option) | Equity (Implicit call option) |

**Public Sector Balance Sheet**

| Assets | Liabilities |
|---|---|
| Foreign Reserves | Financial Guarantee (Implicit put option) |
| Net Fiscal Asset and Other Assets | Foreign Debt (Default-free value of debt minus implicit put option) |
| Value of Monopoly on Issue of Money | Base Money and Local-currency Debt (Implicit call options) |

These three economic balance sheets demonstrate the interdependence among sectors; with one sector "long" a certain implicit option and another sector "short" the same implicit option. This shows how implicit options create important inter-linkages between different sectors. A second type of inter-linkage is through the impact of changes in prices - exchange rate, interest rate, other prices - on default barriers and assets across the various sectors. The default barrier in the various sectors can change due to changes in exchange rates, capital flows, interest rates and other factors. The economic balance[21] sheets for each of the three sectors with illustrative numbers are shown in Annex 5.

## Integrated Value and Risk Transmission between Sectors

The framework described above is versatile and can be used to understand many types of crises and risk shifting that cannot as easily be analyzed with other techniques. The risk-transmission patterns can be dampened or may be magnified depending on the capital structure and linkages. The framework can help identify situations where volatility gets magnified and negative feedback loops that can trigger severe crises. The patterns of value and default correlation across different asset classes, sectors and sovereign debt values depend on these structures and links, unique to a particular economy. Below are some examples of risk transmission between sectors. Actual risk transfer in an economy is likely to involve several risk-transmission channels.

### Risk Transmission from the Corporate Sector to the Banking Sector and to the Government

The corporate sector's financial distress – possibly caused by stock market declines which reduce the value of corporate assets, recession, commodity price drops, or excessive unhedged foreign debt accompanied by currency devaluation – can be transmitted to the financial sector.

[21] The economic balance sheet is the "mark-to-market" balance sheet of the sector's assets and liabilities, including the economic values of other relevant contingent assets and contingent liabilities. This is in contrast to a traditional GAAP accounting balance sheet. For example, the government financial guarantee to the banking system is not a GAAP entry.

*Risk Transfer*

**Corporate Sector** → **Banking Sector** → **Government**

We can use the three-sector framework to show how the risk can be transmitted from the corporate sector to the banking sector and to the public sector through implicit and explicit guarantees. The example of a negative shock to the corporate sector is a drop in assets as a result of recession, equity sell-offs, the combination of currency devaluation and unhedged foreign debt, or other negative shocks (shown in Annex 5, Figure A5-2). The value of the assets of the corporate sector declines, so does the value of the debt (and equity) which leads to a decline in bank assets and an increase in the implicit government guarantee. As the corporate assets decline, the government guarantee to the banking sector increases in a non-linear way (see Figures 5 and 6).

## Figure 5

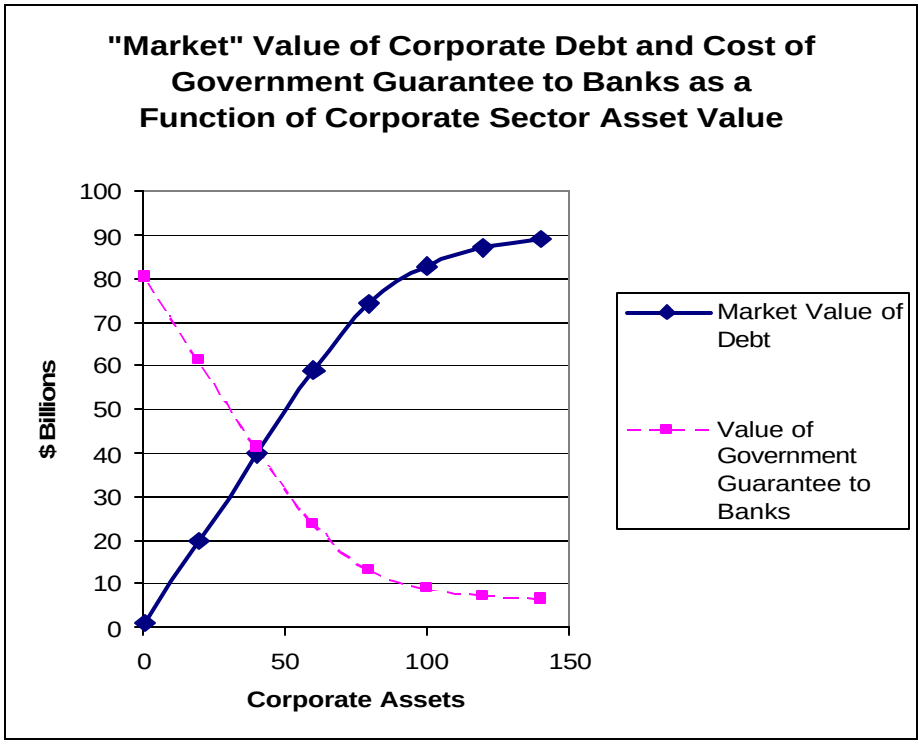
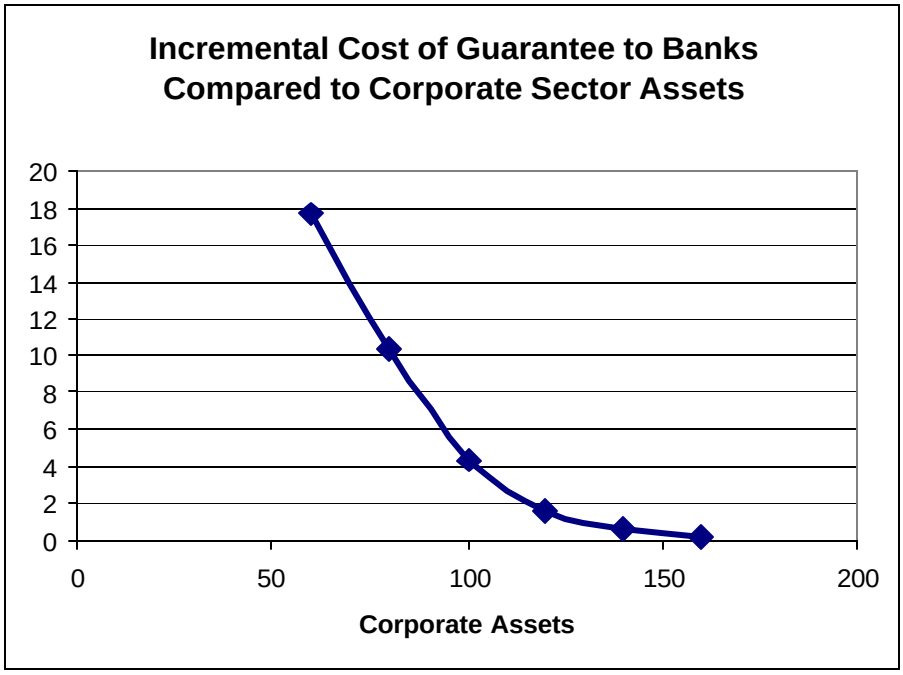


**Figure 6**




**Risk Transmission from the Banking Sector to the Government**

The banking sector's financial distress, such as systemic banking crises due to deposit runs, a decline in asset value or mismanagement, can be transmitted to the government through guarantees.

*Risk Transfer*

**Banking Sector** → **Government**

The example of the impact of a deposit run on the balance sheets of the three-sector model in Annex 5, Figure A5-3 shows that the banking sector's default barrier rises, when the banking sector faces a large deposit run resulting in a large increase in the implicit guarantee.

**Risk Transmission from the Government to the Banks and Feedback**

The public sector's financial distress or default can transmit risk to the financial system. When the banking sector is holding a significant proportion of government securities, and there is a negative shock to the government financial position, it can have

a detrimental impact on the banks. The government's implicit guarantee is also likely to increase. This, in turn, makes the government financial position worse, creating a compounding effect, which may result in the government's failure to honor its guarantee obligations and cause a collapse of the banking system.

*Risk Transfer*

**Banking/Financial System** ⇄ **Government**

Figure A5-4 in Annex 5 shows the impact of a decline in government assets resulting in lower value of sovereign debt relative to the base case. In Figure A5-5, we assume the same decline in government assets but that the banking sector's assets consist of *half* government securities and *half* loans to the corporate sector (as compared to 100% corporate loans in the base case). The vicious cycle could arise, when the lower value of government securities lowers bank assets, and raises the implicit financial guarantee, which in turn lowers government assets further. This means that the implicit guarantee is higher than what is shown above. In some situations, this vicious cycle can spiral out of control, eventually resulting in the inability of the government to provide sufficient guarantees to banks and leading to a systemic financial crisis.

**Risk Transmission from the Pension System to the Government**

The financial distress related to pension plans can result in the transmission of risk to the government.

*Risk Transfer*

**Pension System** → **Government**

Figure A5-6 shows an example of this type or risk transmission. We assume that the pension system contains one-half of corporate sector equity (in a defined benefit plan which has an implicit government guarantee). A decline in corporate assets would cause the corporate equity value to drop. This, in turn, increases the government guarantee to the pension system and the implicit guarantee to banks.

**Risk Transmission from the Public Sector to Holders of Public Sector Debt**

Fiscal, banking and other problems can cause distress for the government which can transmit risk to holders of government debt.

*Risk Transfer*

**Public Sector: Fiscal, Banking, Other** ⟶ **Debt Holders (**e.g. holders of external debt)

Holders of foreign-currency debt have a claim on the value of the debt minus the potential credit loss, which is dependent on the level of assets of the public sector (in foreign currency terms) compared to the foreign-currency default barrier. Thus, we can use the CCA approach to analyze the value of public sector foreign-currency debt by comparing how the volatility of the public sector assets (measured in US dollar terms) changes relative to the foreign-currency default barrier.

A large component of the spread on sovereign foreign-currency debt is the credit spread to compensate for the risk of default over the horizon. The credit spread on sovereign foreign-currency debt is a function of: (i) the ratio of sovereign asset, A, to the default barrier, $DB_F$ (associated with default free debt value of foreign debt; (ii) the volatility of sovereign assets, $\sigma_A$; and, (iii) horizon and risk-free interest rate. [22] As the term ($A/ DB_F$) declines and/or $\sigma_A$ increases, the spread increases in a *non-linear* way and eventually turns sharply higher. The total public sector asset includes foreign currency reserves, the net fiscal asset, and the value of seigniorage in US dollar terms. Thus a decline in foreign currency reserves (or contingent reserves), lower fiscal revenues, and/or a rise the foreign debt default barrier will raise spreads. The volatility of the public sector asset is heavily influenced by exchange rate and fiscal volatilities. In the crisis periods, the fiscal volatility and exchange rate volatility can combine to produce a higher volatility of the sovereign asset $\sigma_{FA}$ [23] (measured in US dollar terms) and thus increase sovereign spreads. See Annex 6 for a more in-depth discussion of sovereign debt valuation and applications to debt sustainability.

[22] Spread = $- 1/T \ln[ N(d_2) + (A/ (DB_F e^{-rt})) N(-d_1)]$, see Annex 1.

[23] The relationships between local currency fiscal asset volatility, exchange rate volatility and correlation is: $({}_{LC}\sigma^2_{FA} + \sigma^2_{ER} - 2 \rho_{ER,FA} \sigma_{ER\ LC}\sigma_{FA})^{1/2} = \sigma_{FA}$

## Interrelationship of Macro Financial Contingent Claim Balance Sheets, Risk Exposures and Traditional Macroeconomic Flows

The proposed macro financial risk framework shows how to construct the balance sheets of key sectors. We can estimate the risk exposures of the implicit option values which respond to changes of the assets and liability structures and asset volatility. There may also be information on explicit derivatives (e.g. central bank explicit derivatives). By using risk management tools to analyze risk exposures from *both* implicit options and explicit derivatives, we can greatly improve risk and policy analyses.

The combined accounts – income/flow, mark-to-market balance sheets, and risk exposure measures – comprise the three important sets of interrelated accounts in the economy which are somewhat similar to those in large modern financial institutions. Risk managers would find it difficult to analyze the risk exposure of their financial institution by relying solely on the income and cash flow statements, and not taking into account (mark-to-market) balance sheets or information on their institution's derivative or option positions. The country risk analysis that relies only on macroeconomic flow-based approach is deficient in a similar way; the traditional analysis does not take into account the volatility of assets.

We can derive something similar to the traditional macroeconomic income accounts from a specific, static case of the general macrofinancial contingent claim equations. Since traditional macroeconomic flow-of-funds accounts typically look at changes over a specific time period (such as a year), one can compare the change in value of the CCA assets and liabilities of different sectors over a year and see how this is related to the traditional macroeconomic flow-of-funds over the same period. Beginning with the macrofinancial CCA equations for the main sectors, if one assumes the volatility of assets equals zero, then the put options and guarantees would equal zero. In this situation, the value of debt is no longer the market value, but the default-free value. Thus, the change in income from one period to the next would then equal the change in default-free value of debt plus the change in foreign reserves and local currency. These are very similar to the public and private sector flow equations in the flow-of-funds accounts. See Annex 7 for additional details.

## Types of Crises

The CCA-based framework can help analyze value changes and various channels of risk transmission between sectors in the emerging markets and the developed economies.

*Emerging Market Countries* - Large capital inflows tend to coincide with a rise in the equity values of the corporate and financial sectors, leading to an increase in the implied value of their assets. In this case, the entities can roll over their short-term debt without difficulties. However, when the capital flows reverse, equity and asset prices tend to fall simultaneously while default barriers may rise. The asset values and default barriers in key sectors diverge with capital inflows and converge with outflows.[24] Currency appreciation in the first phase and depreciation in the second phase can compound these effects. Joint currency, financial and debt crises are increasingly common and are usually linked to governments having insufficient foreign-currency assets to maintain a particular level of the exchange rate and to provide financial support to the financial system.

Countries with large amounts of foreign-currency debt in the corporate and banking sectors such as Thailand and Indonesia in 1997 experienced a devastating feedback loop as a large currency devaluation caused corporate and banking crises, which required large government interventions. Subsequently, the financial position of the government worsened, further fueling the currency crisis.

Argentina (2001-2) is a case where the decline in public sector assets triggered a default on domestic and foreign debt, resulting in actions leading to a severe banking and currency crisis. Brazil has sizable foreign-currency-linked and local-currency debt. In 2002, political and economic uncertainty caused a significant jump in the public sector asset volatility and spreads on debt along with a substantial devaluation. Later the spreads declined in tandem with lower asset volatility in the public sector.

*Developed Countries* - Japan is a case of combined banking, corporate, and pension funding crises with important exchange rate and fiscal linkages. The proposed framework is particularly suitable for the analysis of these complex interrelated problems. The deteriorating financial position of firms in the US has a negative impact on the

banking system and reduces their ability to contribute sufficient resources to fund their pension obligations. The Pension Benefit Guarantee Corporation has a large and growing contingent obligation which can be analyzed with the framework laid out here.

## Controlling and Transferring Risk

The application of modern financial engineering to analyze risk exposures in the sectors of an economy offers a rich framework for comparing alternative ways to control and transfer risk. There are several benefits of this approach. First, it gives the interrelated values and risk exposure measures across sectors. Understanding of these values and risk exposures can help identify particularly vulnerable situations and potential chain reactions of default. This allows formulation of various alternative ways to control and transfer risk. Second, the framework dovetails with risk-management strategies involving *explicit derivatives and swaps* used by the private and public sectors to control, hedge or transfer risk.[25]

We will touch on a few of the strategies to control, mitigate and transfer risk. Four broad categories of strategies are: a direct change in the financial structure (the structure of assets and liabilities within the existing institutional structure); managing guarantees; risk transfer (diversification, hedging and insurance); and, in the longer run, an institutional change to tailor the institutional structure to fulfill financial functions more efficiently within the specific geopolitical environment.

### Direct Change in Financial Structure

Increases in assets and declines in default barriers can reduce the vulnerability to distress, reduce spreads on debt, and reduce the value and the deltas of put options (whether they are embedded in risky debt or financial guarantees from the government). CCA, by its nature, shows how the changes in value of assets relate to changes in values of liabilities. Thus, it provides a natural framework for analysis of mismatches, such a currency and maturity mismatches on balance sheets. Policies or actions that reduce these mismatches will help reduce risk and vulnerability.

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[24] See Pettis, 2001, for analysis of capital flows and risk in emerging markets.
[25] One example, in Blejer and Schumacher (2000), includes central bank forward contracts.

**Management of Guarantees**

CCA provides the key to measuring the value and understanding the risk of guarantees. The three basic methods that a guarantor of liabilities has to manage the risks of guarantees are:

*Monitoring* – The method requires frequent marking-to-market of assets and liabilities of the insured party and collateral that can be seized when the insured party's assets fall below a predetermined target.

*Asset Restrictions* – This method of controlling costs and managing the cost of the guarantee requires the insured party to (at least partially) hedge its guaranteed liabilities with restrictions on assets in a manner that limits the volatility of net worth.

*Risk-based premiums* – Under this method, the guarantor charges a fee that is commensurate with the riskiness of the guarantee.[26]

Guarantees on the debt of financial institutions, whether explicit or implicit, should be openly recognized in the government's balance sheet. This framework provides a way to measure the value of the guarantee and the risk exposures associated with the guarantee.

**Risk Transfer**

There are three ways to transfer risk, *diversification, hedging and insurance*. Much of the risk described here results from concentration risk and diversification to parties who have a comparative advantage in bearing various risks. If the balance sheets of corporations and financial institutions are weak when the economy is weak—as it is generally the case—then it is precisely when tax revenue is low, and the cost of debt service is high because sovereign risk has increased. In this case, the value of the guarantees will be particularly high. This observation offers a powerful argument for diversification of the government exposure to local shocks (see Box 2).

The financial markets, especially in emerging markets, are often "incomplete", meaning that they provide only limited possibilities to shift risk across various entities and groups. In such situations, diversification through international capital mobility is

[26] See Merton and Bodie (1992, 1993) and Bodie and Merton (1993).

the obvious alternative. However, the transfer across borders of the ownership of real and financial assets is a rather inflexible way to achieve diversification (as it is costly to reverse); often it also runs against political constraints.

**Box 2**

## Examples of Diversification, Hedging and Risk Mitigation

- *Sovereign Bonds with Special Features* – GDP-linked bonds or bonds with specific roll-over clauses can help manage risk.
- *Contingent Purchase of Government Local-Currency Debt* – Corporations sometimes contract for contingent equity purchases triggered under pre-agreed conditions. Similarly, governments could make arrangements with external public or private sector entities for pre-agreed purchase of government local-currency debt under specific circumstances.
- *Asset Diversification in Banking Sector* – Asset diversification would suggest that a bank which invests part of its assets in domestic government bonds enhances its exposure to local macro shocks; the value of government bonds will be low precisely when the value of the loan book is low. Therefore, in such economies, banks should hedge the exposure of their loan book by investing in non-domestic assets—such as bonds.
- *Equity Swaps as a Method of Diversifying Internationally* – An equity swap would enable a small country to diversify internationally without violating possible restrictions on investing capital abroad. Suppose that small-country pension funds who already own the domestic equity were to enter into swaps with a global pension intermediary (GPI). In the swap, the total return per dollar on the small country's stock market is exchanged annually for the total return per dollar on a market-value weighted-average of the world stock markets. The swap effectively transfers the risk of the small-country stock market to foreign investors and provides the domestic investors with the risk-return pattern of a well-diversified world portfolio. Since there are no initial payments between parties, there are no initial capital flows in or out of the country. Subsequent payments, which may be either inflows or outflows, involve only the *difference* between the *returns* on the two stock market indices, and no "principal" amount flow.
- *Capital/Foreign Currency Cushion Against Unexpected Loss* – This macrofinance framework could be used to do value-at-risk for the sovereign balance sheet to help determine the appropriate level of foreign currency reserves or contingent reserves.
- *Others* – Other types of swaps could include assets, equity, or debt of the corporate sector, the financial sector, and the public sector.

The macro finance analytical framework could be useful for the design of new risk intermediation and risk transfer products, whereby various risks in one economy could be packaged and sold internationally to improve the efficiency of risk sharing and enhance returns.

**Institutional Change**

In the longer-term, institutional changes to satisfy certain financial functions more efficiently can reduce risk. In the case of the banking sector, this is particularly important, given the vulnerabilities and costs of crisis in the banking system. Fiscal costs of banking crisis show no sign of declining and can range from costs of 3% to 80% of GDP, not to mention the inefficiencies caused before, during and after the crises. The potential for very costly government guarantees to the banking system, which can arise quickly and can have large associated risk exposures and costs, support the arguments *that it may be best to safely shrink the banking system*.[27] Structural reform, over time, could aim to reduce the size of the banking system and increase the role of institutions that can fulfill the key functions of banks (payments functions and pooling and investment of resources) but do so in an efficient and less risky manner.

The combination of a smaller banking system, improved management of guarantees, equity swaps between the pension system and international counterparties, and direct change in the financial structure would dramatically reduce risk exposures and systemic vulnerability.

## Conclusions

The high cost of international economic and financial crises highlights the need for a comprehensive framework to assess the robustness of countries' economic and financial systems. This paper proposes a new approach to measure, analyze, and manage macroeconomic risk based on the theory and practice of modern contingent claims analysis (CCA). We illustrate how to use CCA to model and measure sectoral and national risk exposures, and we analyze policies to offset their potentially harmful effects. The framework provides economic balance sheets for inter-linked sectors and a risk accounting framework for an economy. It is useful for assessing vulnerability, policy analysis, risk management, investment analysis, and design of risk control strategies. Both public and private sector participants can benefit from pursuing ways to facilitate more efficient macro risk accounting, improved price and volatility discovery, and expanding international risk intermediation activities.

[27] Posen, A. 2001, *"A Strategy to Prevent Financial Crises: Safely Shrink the Banking Sector."*

## Annexes

## Annex 1 - Framework for Contingent Claims Analysis, Risk Measures, and Spreads Using Black-Scholes-Merton Formula

CCA defines these fundamental relationships between the value of assets and the value of claims. The total market value of assets, A(t), of an entity financed with debt and junior claims (most junior claim or equity) is equal to the market value of junior claims and market value of risky debt, J(t) + D(t). The default-free value of debt represents a default barrier, DB. Assets are stochastic and thus assets, at time t, in the future may decline below the point where debt payments on scheduled dates cannot be made. Asset value includes is derived from the stochastic present value of income minus expenditures. If assets fall to or below the level of the DB, then default is the result. The holders of the junior claim have a contingent claim on the residual value of assets in the future, which is the maximum of either assets minus debt, or nothing. So the junior claim, J(t) at T, is J(T) = max [A(T) – DB, 0]. The holders of risky debt receive either the default-free value or, in the case of default, have a claim on assets. Thus, the market value of risky debt can be modeled as D(T) = min [A(T), DB] = DB – max [DB – A(T),0]. The value of assets of the entity at time T is:

Assets = Junior Claim + Risky Debt = Junior Claim
+ Default-Free Debt – Debt Guarantee
= A (T) = J(T) + D(T)
= max [A(T) – DB, 0] + DB – max [DB – A(T),0]

The value of the junior claim, the debt guarantee embedded in the value of risky debt, and the financial guarantee can all be formulated in terms of *implicit options* (Merton, 1974).

Junior Claim/Equity = Implicit Call Option = max [A(T) – DB, 0]

Risky Debt = Default-Free Debt – Debt Guarantee
= Default-Free Debt – Implicit Put Option = DB – max [DB – A(T),0]

Financial Guarantee = Implicit Put Option = max [DB – A(T),0]

In the case of risky debt, the debt holder is offering an implicit guarantee as it is obligated to absorb the losses if there is default. In this case the lender and the guarantor are the same entity. This implicit guarantee is modeled as an implicit put option. If, however, a third party is the guarantor (e.g. the government), the value of the financial guarantee is an implicit put option as well. Examples are government financial guarantees for banks or other entities.

**Black-Scholes-Merton Formula**

Using the Black-Scholes-Merton formula for the implicit call and implicit put options:

$$J = N(d_1)\,A - N(d_2)\,DB\,e^{-rT}$$
$$d_1 = \{\ln(A/DB) + (r + \sigma^2/2)T\} / \sigma T^{1/2}$$
$$d_2 = d_1 - \sigma T^{1/2}$$

and, P = Implicit Put Option = max [DB – A(T),0]

$$P = N(-d_2)\,DB\,e^{-rT} - N(-d_1)\,A$$

Where:

A = value of the assets
J = value of the most junior claim
DB = default barrier or default-free debt value
r = risk-free interest rate
T = time to maturity of the debt in years
$\sigma$ = standard deviation of the annualized continuously compounded rate of return on the assets
N(d) = the probability that a random draw from a standard normal distribution will be less than d.

Credit spread (over a benchmark interest rate) on the debt is:

$$\text{Spread} = -1/T \ \ln\left[N(d_2) + (A/(DB\,e^{-rt}))\,N(-d_1)\right]$$

This formula is derived from the spread needed to compensate for the expected loss associated with the value of the put option (value of implicit guarantee).
The *probability of default*, using this model, is $N(-d_2)$.[28]

**Example:** Assuming that:

Asset value A=$100,
Asset return volatility of $\sigma$ = 0.40 (40%),
Default-free value of debt = default barrier = DB = $ 75
(derived from short-term debt, $30, plus one-half of long-term debt, $90)
Risk-free rate = 0.05 (5%)
Time horizon = 1, one year

The value of the junior claim/equity is $32.2 and the value of risky debt is $67.8 (equal to the present value of the default barrier minus put option = 75*0.95 – 3.55 = 67.8). The "delta" for the call option is $N(d_1)$, is 0.89 in the above example. The "delta" for the put option (implicit guarantee) is $N(d_1) - 1$, or -0.11 in the example. Using the spread formula above, the one-year spread for the example is calculated as: 0.0510, or 510 basis points over the risk-free rate. The *probability of default*, using this model, is $N(-d_2)$, or 0.20 (20%) in the example above.

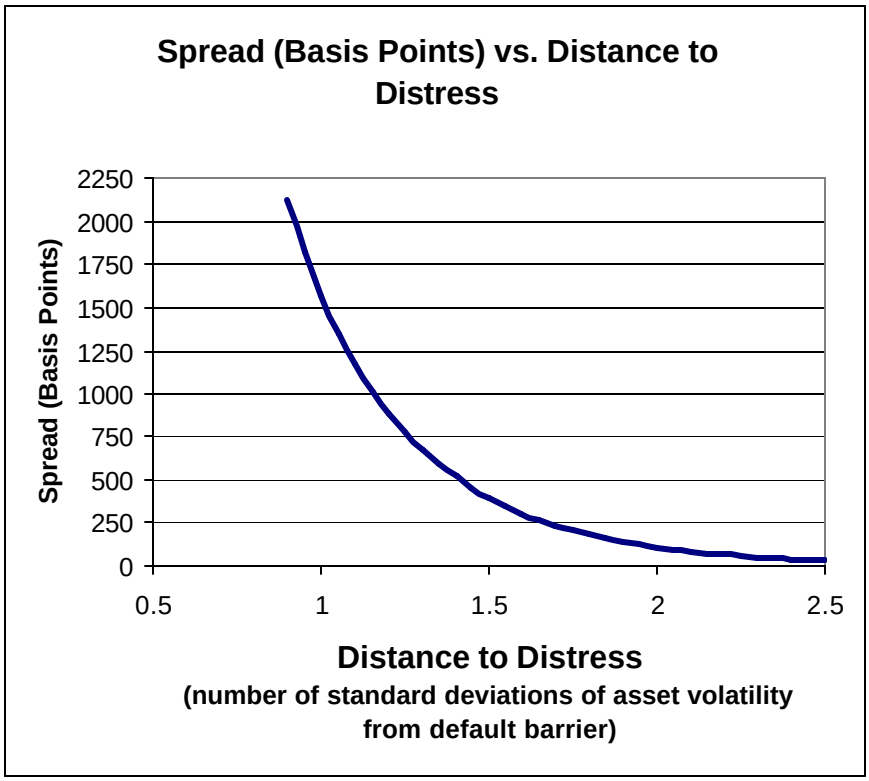
[28] For a recently published book explaining these concepts applied to credit risk, see Crouhy, Galai, and Mark, 2000.

**Distance-to-Distress and Credit Spreads**

The "distance to distress" is a useful indicator of default risk. It is essentially the asset value minus the default barrier normalized by the asset volatility. Distance to Distress = (Expected Asset Value - DB) / (Asset Value)*(volatility of Asset).

Figure A1-1 below shows a graph of the credit spread compared to the distance-to-distress. The nonlinear relationship shown by the curve is an important risk measure. The distance to distress declines as assets fall relative to the default barrier and/or when volatility of assets increases.

**Figure A1-1**



## Annex 2 - Money on the Public Sector CCA Balance Sheet

This Annex describes the value money on the asset side of the balance sheet, derived from the public sector monopoly on issue of money, and the value of money to the holders for money for liquidity and convenience as well as the additional value of money associated with the option to exchange it for foreign currency.

**Public Sector Monopoly on Money Issue**

Money on the public sector balance sheet (combined government and monetary authorities) has components on both the liability side and the asset side of the balance sheet. Base money (also known as reserve money or high-powered money) is a liability on the public sector balance sheet. The asset side of balance sheet includes the benefits associated with the public sector monopoly on the issue of money. As described by Buiter (1993), the stock of high-powered money is a liability on the public sector balance sheet and net value of the government's cash monopoly is on the asset side of the public sector balance sheet. He defines the imputed net value of the government's cash monopoly:

$$A^M(t) = \frac{1}{p(t)} \int_t^{\infty} \hat{i}(u,t)\, d\hat{H}(u,t)\, dt e^{-\int \hat{i}(s,t)\, ds}\, du$$

Where $\hat{i}(u,t)$ is the expected value of nominal interest rates at time t to prevail at time u, and $\hat{H}(u,t)$ is the expected value of high-powered money (base money) at time t to prevail at time u. The price level is *p(t)*. Buiter takes the discounted flow of profits from the government operating the printing presses and derives the equations above, assuming that the creation of money is costless. He interprets this imputed net value of the government's monopoly on money as equivalent to the present discounted value of interest income expected to be earned at each future date on a portfolio of government bonds that is equal in value to the stock of high-powered money at that date. This is equal to the imputed present value of seigniorage (Blejer and Cheasty, page 285).[29]

**Liquidity and Convenience Value of Money to the Holders of Money**

Money is a liability on the public sector balance sheet. Its real value is equal to H/p, as described below. Money has value to the holders of money because it yields a "flow of imputed nonpecuniary liquidity and convenience services" according to Buiter 1993, who describes the value to the holders of money at time t equal to $V^M(t)$ which is equal to:

$$V^M(t) = \frac{1}{p(t)} \int_t^{\infty} dH(u,t)\, \hat{p}^M(u,t) e^{-\int \hat{i}(s,t)\, ds}\, du$$

[29] Seigniorage for a given period is defined as the change in money divided by the price level, the monetary authority portion of seigniorage, $S_t$, is associated with the change in base money, M: $S_t = (M_{t+1} - M_t)/P_t$.

The term $p^M$ is the non-pecuniary return on money. The assumption that the pecuniary and nonpecuniary yields money and bonds are equalized at the margin (ignoring tax effects) yields $p^M = i$. Both equations together for $V^M(t)$ above implies that $V^M(t) = H/p$.

There is an extensive literature on approaches to valuing the liquidity and convenience yield of money which will not be described in detail and is not central to the framework described here. A useful summary is provided by Walsh (2001),

**Value of the Option to Exchange Money for Foreign Currency**

In addition to the imputed liquidity and convenience yield of money, money has value (to the holders of local currency) derived from the "option" to exchange the local currency for foreign currency. From the perspective of the economy as a whole, the exchange of money for foreign currency results in a change in the net foreign reserves of the banking system which results in a one-for-one reduction in the foreign currency reserves of the monetary authorities (described in IMF 2000 and Schaechter, 2001). The holders of money have the option to either hold money for its liquidity and convenience yield or exchange it for foreign currency which results in a reduction of the foreign currency reserves of the public sector (if there are no capital controls or restrictions on such an exchange). Holders of money thus have an option to exchange one asset, local currency, for another asset, residual foreign reserves (foreign reserves after obligations to supply foreign currency to pay senior obligations such as foreign-currency debt). The option to exchange one asset for another is max [asset 1, asset 2].[30]

The holders of money are "long" a call option on the public sector foreign currency reserves that can be exchanged for local currency, i.e. a claim on the residual foreign reserves left over after higher priority claims on foreign currency reserves have been made, such as payment of foreign debt (max [FX reserves – FX obligations to pay FX Debt, 0]). The public sector has the opposite position to the holders of money; it is "short" the call option associated with the issue of base money.[31] Capital controls restricting or preventing such an exchange will limit the reserves available for the exchange. Such a situation is most likely to occur when reserves have been used for higher priority uses.

[30] This is equivalent to asset 1 + max [asset 2 – asset 1, 0 ].

[31] Note also that, in some situations, base money is exchanged for residual foreign reserves when an economy is 'dollarized'. For example, in Ecuador, the crises sequence included a deposit freeze, restructuring of local debt, default on foreign debt, hyperinflation and then dollarization where base money was exchanged for a portion of residual foreign reserves at a very depreciated exchange rate.

## Annex 3 – Public Sector CCA Balance Sheet

This Annex describes a framework for the segregated contingent claim balance sheet of the monetary authorities and the government with a separation of the local currency activities and foreign currency activities. In a corporate parent-subsidiary structure, the parent entity typically holds equity of the subsidiary as an asset side of its balance sheet and guarantees to the subsidiary on the liability side. Useful insights can be obtained when one views relationship between the assets and liabilities of the public sector in a similar way as separate balance sheets of the government and monetary authorities where there are cross-holdings and financial guarantees between these two public sector "partners." Under this structure, the assets of the monetary authority include foreign reserves and claims on government local-currency debt and other obligations. The liabilities of the monetary authority partner are base money (specifically the obligation to exchange outstanding base money for foreign currency) and financial guarantees to the government, including guarantees to supply foreign currency to service the sovereign foreign-currency denominated debt. The assets of the government partner include the net fiscal asset, the present value of seigniorage, and other assets, while the liabilities include local-currency debt held by the monetary authority, local-currency debt held outside of the monetary authority, financial guarantees and foreign debt.

Figure A3-1 shows the structure of this segregated or "unconsolidated" public sector structure. This simplified framework is not meant to be a comprehensive catalogue of all the guarantees, the nature of which varies by country and by the detailed structure of the relationship between monetary authorities and the government. There may be also be a "guarantee" of the monetary authorities to buy government local-currency debt under certain circumstances, but this is not shown here. The action of the monetary authority "partner" of buying additional local-currency debt entails issue of additional base money. There are also "options," that the government and the monetary authorities have, to "default" on the obligations to convert local currency into foreign currency and to "forcibly" restructure local-currency debt or to dictate "mandatory" purchases of government bonds by certain public or private institutions. Also, in some countries, banks may have deposits with the monetary authorities that receive a higher priority claim on foreign currency reserves than the holders of local currency, which could be junior to claims on foreign currency for payment of external foreign-currency debt.[32]

[32] The authors would like to thank Sean Craig at the IMF, David Levey, Dan Russell, Simon Jiang, Praveen Varma and Chris Mann at Moody's Investor's Service for useful comments on these issues.

## Figure A3-1 – Segregated Balance Sheet for the Public Sector

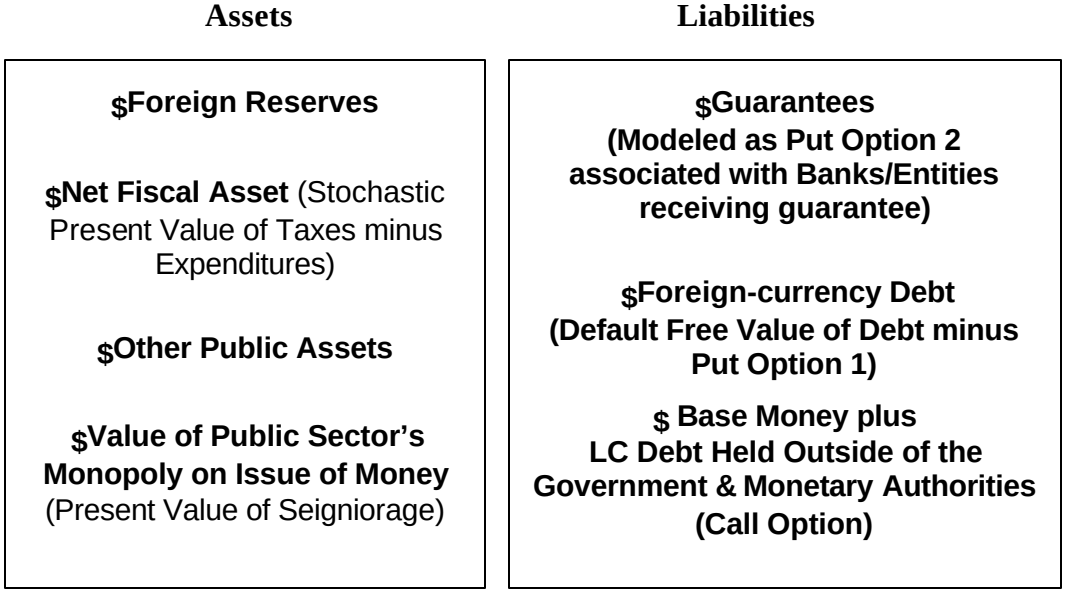
| Assets | Liabilities |
|---|---|
| **MONETARY AUTHORITY "PARTNER"** | |
| **Foreign Reserves**<br><br>**Contingent Foreign Reserves**<br><br>***Local-currency Debt Held by Monetary Authorities and Other Credit to Government*** | **Base Money (Obligation to exchange base money for foreign currency)**<br><br>***Obligation to supply FX to Government to pay FX Debt*** |
| **GOVERNMENT "PARTNER"** | |
| **Local Currency Operations of Government** | |
| **Net Fiscal Asset**<br><br>**Other Public Assets**<br><br>**Value of Public Sector's Monopoly on Issue of Money** | ***Local-currency Debt Held by Monetary Authorities and Other Credit to Government***<br><br>**Local-currency Debt Held Outside of the Government & Monetary Authorities**<br><br>**Guarantees (to too-important-to-fail entities)** |
| **Foreign-currency Debt Operations of Government** | |
| ***Obligation from Monetary Authority "Parent" to supply FX to Government to pay FX Debt*** | **Foreign-currency Debt** |

Note that the cross-holdings of government debt and guarantees from monetary authorities to Government are in italics. Liquidity operations of the monetary authorities are not included.

The local-currency debt held by the monetary authorities is an asset on the side of the monetary authority partner and a liability of the government partner. Similarly, the financial guarantees to the government partner are an asset on its balance sheet and a liability of the monetary authority partner. When the balance sheets are combined these two items drop out. The segregated parent-subsidiary balance sheet above reduces to the combined balance sheet in Figure 1. Contingent claims approach can be applied to the segregated or the combined balance sheets, the choice of which depends on the purposes of the analysis.

The option to exchange money for foreign currency reserves, associated with base money, can be viewed as a call option on foreign currency reserves less the guarantees to supply foreign currency for higher priority purposes like external debt payments, as described in Annex 2. The combined claim of base money and local-currency debt held outside the monetary authorities is a claim on public sector assets (foreign currency reserves *minus* guarantees to supply FX for external debt *plus* net fiscal asset plus other public assets *plus* present value of seigniorage *plus* guarantees to supply FX for external debt) with the default barrier being the senior public sector liabilities (foreign-currency debt and guarantees to too-important-to-fail entities). On a simplified combined balance sheet, the local-currency debt of the government, held outside of the monetary authorities, and base money can be modeled as a call option on the public sector assets with the default barrier being senior liabilities – foreign-currency debt and guarantees to too-important-to-fail entities. See Figure A3-2 below.

## Figure A3-2 Example Public Sector Contingent Claims Balance Sheet with Liabilities Modeled with Options (all items in $ terms)




The units in which the balance sheet is measured can be nominal local currency units, in real terms in local currency units, or in foreign currency. Any numeraire can be used. Since one of the goals of the analysis here is to analyze the value of the CCA balance sheet in the international context (including effects of changes in FX reserves and the credit risk embedded in foreign debt), the units are in US $.

## Annex 4 - Obtaining Implied Asset and Asset Volatility Values from Market Information

The market value of assets and asset volatility by sector or for the public sector are not directly observable. These two values can be estimated based on observed values and volatilities of other components of the contingent claims formulas. Using the Black-Scholes-Merton option pricing model along with another mathematical tool that relates the asset volatility to equity and equity volatility, it is possible to solve for the *implied* [33] market value of assets and asset volatility. Once the implied values for assets and asset volatility are obtained, it is possible to construct the market value CCA balance sheets for corporate sub-sectors, the public sector and as input to the balance sheets of banking and financial sub-sectors.

The following section briefly describes how the values implied asset and asset volatility are obtained. The market value of junior claims (equity in the case of firms) is represented as a call option on the assets:

$$J = \text{Call Option} [ \mathbf{A}, s_{\mathbf{A}} , DB(T), r, T] \qquad \textit{[Eq. 1]}$$

It can also be shown that the following relationship holds:

$$\sigma_J = s_{\mathbf{A}} * \mathbf{A} * N(d_1) / J \qquad \textit{[Eq. 2]}$$

*Where*:

- $A$ = ***Market value of assets at time 0***
- $s_A$ = ***Volatility of assets***
- $DB(T)$ = *Default barrier*
- $r$ = *Risk-free rate of interest*
- $T$ = *Time*
- $\sigma_J$ = *Volatility of most junior claim or equity*
- $J$ = *Market value of most junior claim or equity at time 0*
- $N(d_1)$ = *Change in the junior claim value divided by the change in the asset value (see Annex 1 for formula)*

Both unknowns have been highlighted in bold and, with the two equations, both unknowns can be solved through an iterative process.

[33] An implied value refers to an estimate derived from other observed data. Techniques for using implied values are widely practiced in options pricing and financial engineering applications.

## Annex 5 – Examples of Risk Transmission Channels between Economic Balance Sheets of Sectors

This Annex describes the CCA equations and inter-linkages among sectors for a simple three sector framework and numerical examples for a number of different scenarios which demonstrate value and risk transmission and calculation of risk exposures.

Figures in Annex 5:

1. Base case sectoral balance sheet.

(Changes in parentheses in all figures are all relative to base case)

2. Negative shock to corporate sector assets and the subsequent impact on banking assets and increase in government implicit guarantee.

3. Deposit run and subsequent rise in government implicit guarantee.

4. Negative shock to government assets resulting in lower value of sovereign debt.

5. Negative shock to government assets and thus a decline in the value of government securities held by banks leading to an increase in implicit financial guarantee (and feedback loop).

6. Negative shock to corporate sector assets (as in 2. above) in the case where the pension system contains one-half of corporate sector equity (in a defined benefit plan which also has an implicit government guarantee). The negative shock to corporate sector assets results in lower pension system's asset value with higher implicit guarantee for pension system, in conjunction with higher implicit guarantee to banks.

## Illustrative Equations for Inter-linked Sectors

*MV stands for market value, A is assets, E is equity, J is junior claim, D is debt, FS is financial guarantee from government, C means call option, P means put option. (Subscripts C, B, and G refer to the corporate, banking and financial, and public sector, respectively.)*

### *Corporate Sector*

Market value balance sheet equation for the corporate sector(s) using contingent claims.

$MV(A_C) = MV(E_C) + MV(D_C)$ *[Market Value Balance Sheet Equation]*

$MV(D_C) = DB_C - P[MV(A_C), DB_C, \sigma_C, T, r]$ *[ Equation for $MV(D_C)$]*

$MV(A_C) = C[MV(A_C), DB_C, \sigma_C, T, r] + DB_C - P[MV(A_C), DB_C, \sigma_C, T, r]$

### *Bank and Financial Institutions Sector*

Using the market value balance sheet equation, we substitute variables representing the contingent claims:

$MV(A_B) + FS_G = MV(E_B) + MV(D_B)$ *[Market Value Balance Sheet Equation]*

$FS_G = P[MV(A_B), DB_B, \sigma_B, T, r]$ *[Contingent Financial Support, i.e. Guarantee]*

$MV(A_B) + FS_G = C[MV(A_B), DB_B, \sigma_B, T, r] + MV(D_B)$

Banking sector assets contain loans to the corporate sector

$MV(D_C) = DB_C - P[MV(A_C), DB_C, \sigma_C, T, r]$ *[Loans Made to the Corporate Sector]*

### *Public sector (Government and Monetary Authority) Sector*

Using the market value balance sheet equation, we substitute in variables representing the contingent claims:

$MV(A_G) = MV(J_G) + MV(D_G) + FS_G$ *[Market Value Balance Sheet Equation]*

$MV(A_G) = C[MV(A_G), DB_G, \sigma_G, T, r] + DB_G - P[MV(A_G), DB_G, \sigma_G, T, r] + P[MV(A_B), DB_B, \sigma_B, T, r]$

$FS_G = P[MV(A_B), DB_B, \sigma_B, T, r]$ *[Contingent Financial Support to Banks/Financial Institutions]*

$MV(D_G) = DB_G - P[MV(A_G), DB_G, \sigma_G, T, r]$ *[Equation for $MV(D_G)$]*

## Annex 5 (cont.) Figure A5-1

Example sectoral economic balance sheets base case:

**Corporate Sector Balance Sheet**

| Assets | | Liabilities | |
|---|---|---|---|
| Corporate Assets | 120 | Loans ( Default-free value = 90, minus implicit loan guarantee – or put option – of 2.8) | 87.2 |
| | | Corporate Equity (call option) | 32.8 |
| Total | 120 | | 120 |

**Banking Sector Balance Sheet**

| Assets | | Liabilities | |
|---|---|---|---|
| Loans (90 -2.8) | 87.2 | Deposits | 81.3 |
| Financial Guarantee | 7.4 | Equity | 13.3 |
| Total | 94.6 | | 94.6 |

**Public Sector Balance Sheet**

| Assets | | Liabilities | |
|---|---|---|---|
| Domestic assets | 100 | Financial Guarantee to Banks | 7.4 |
| Foreign reserves | 40 | Foreign Debt | 82.15 |
| | | Local-currency Debt & Other Liabilities | 50.45 |
| Total | 140 | | 140 |

(Units are in Billions of $)

The *delta* of the guarantee is -0.35 in this base case.

## Annex 5 (cont.) - Figure A5-2

Negative Shock to Corporate Sector Assets, decline of $40 billion (from $120 to $80 billion) as compared to base case.

**Corporate Sector Balance Sheet**

| Assets | | Liabilities | |
|---|---|---|---|
| Corporate Assets | 80<br>(- 40) | Loans ( Default-free value = 90, minus implicit loan guarantee – or put option of 15.8) | 74.1<br>(- 13.1) |
| | | Corporate Equity | 5.9<br>(- 26.9) |
| Total | 80<br>(- 40) | | 80<br>(- 40) |

**Banking Sector Balance Sheet**

| Assets | | Liabilities | |
|---|---|---|---|
| Loans (90 -15.8) | 74.1<br>(- 13.1) | Deposits | 81.3 |
| Financial Guarantee | 13.3<br>(+ 5.7) | Equity | 6.1<br>(-7.2) |
| Total | 87.4<br>(- 7.2) | | 87.4<br>(- 7.2) |

**Public Sector Balance Sheet**

| Assets | | Liabilities | |
|---|---|---|---|
| Domestic assets | 100 | Financial Guarantee to Banks | 13.3<br>(+ 5.7) |
| Foreign reserves | 40 | Foreign Debt | 80.4<br>(- 1.75) |
| | | Local Currency Debt & Other Liabilities | 46.2<br>(- 4.25) |
| Total | 140 | | 140 |

(Units are in Billions of $)

*Risk Transmission*: Lower corporate assets → lower corporate equity and debt → lower bank assets → higher guarantee from government required → higher cost of government guarantee lowers value of sovereign debt. The value of the assets of the corporate sector declines, so does the value of the debt (and equity) which leads to a decline in bank assets and an increase in the implicit government guarantee. The *delta* of the guarantee is -0.56 in this case, as compared to -0.35 in the base case.

## Annex 5 (cont.) - Figure A5-3

Deposit Run - Default barrier rises for banks as $36 billion of long term savings and time deposits become short-term liabilities with deposit run. In this contingent claims framework for banks, the default barrier for banks includes deposits calculated as default-free value of short and long-term deposits which is approximated by demand deposits plus a fraction of time and saving deposits. In a deposit run, a portion of the long-term time and savings deposits shift to the short-term category, thus raising the overall default barrier and raising the size of the implicit guarantee of the government. As $36 billion of long term savings and time deposits become short-term liabilities, the result is a significant increase in implicit financial guarantees from $7.4 billion to $32.6 billion, an increase of $25.2 billion from the base case. The *delta* of the guarantee is -0.83 in this case, more than double the -0.35 delta value in the base case.

**Corporate Sector Balance Sheet**

| Assets | | Liabilities | |
|---|---|---|---|
| Corporate Assets | 120 | Loans ( Default-free value = 90, minus implicit loan guarantee – or put option – of 2.8) | 87.2 |
| | | Corporate Equity | 32.8 |
| Total | 120 | | 120 |

**Banking Sector Balance Sheet**

| Assets | | Liabilities | |
|---|---|---|---|
| Loans (90 -2.8) | 87.2 | Deposits | 117.3 (+ 36) |
| Financial Guarantee | 32.6 (+ 25.2) | Equity | 2.5 (- 10.8) |
| Total | 119.8 (+ 25.2) | | 119.8 (+ 25.2) |

**Public Sector Balance Sheet**

| Assets | | Liabilities | |
|---|---|---|---|
| Domestic assets | 100 | Financial Guarantee to Banks | 32.6 (+ 25.2) |
| Foreign reserves | 40 | Foreign Debt | 73.7 (- 8.4) |
| | | Local-currency Debt & Other Liabilities | 33.7 (- 16.8) |
| Total | 140 | | 140 |

(Units are in Billions of $)

## Annex 5 (cont.) Figure A5-4

Negative Shock to Public Sector Assets of $20 billion, result is lower "market value of liabilities" as compared to base case.

**Corporate Sector Balance Sheet**

| Assets | | Liabilities | |
|---|---|---|---|
| Corporate Assets | 120 | Loans ( Default-free value = 90, minus implicit loan guarantee – or put option – of 2.8) | 87.2 |
| | | Corporate Equity | 32.8 |
| Total | 120 | | 120 |

**Banking Sector Balance Sheet**

| Assets | | Liabilities | |
|---|---|---|---|
| Loans (90 -2.8) | 87.2 | Deposits | 81.3 |
| Financial Guarantee | 7.4 | Equity | 13.3 |
| Total | 94.6 | | 94.6 |

**Public Sector Balance Sheet**

| Assets | | Liabilities | |
|---|---|---|---|
| Domestic assets | 90<br>(- 10) | Financial Guarantee to Banks | 7.4 |
| Foreign reserves | 30<br>(- 10) | Foreign Debt | 79.1<br>( - 3.1) |
| | | Local Currency Debt & Other Liabilities | 33.5<br>( - 16.9) |
| Total | 120<br>(- 20) | | 120<br>(- 20) |

(Units are in Billions of $)

*Risk Transmission*: Negative shock to government assets → lower value of foreign debt (senior debt in this example) and lower value of local-currency debt and other liabilities.

## Annex 5 (cont.) Figure A5-5

Negative Shock to Government Assets and Decline in Value of Government Securities Held by Banks - Banking sector assets consist of half government securities and half loans to corporate sector (as compared to 100% corporate loans in the base case). The market value of government securities (local-currency debt) declines due to decline in government assets of $20 billion relative to the base case. The decline in government assets of $20 billion increases the guarantee to banks by $6.2 billion to $13.6 billion. The vicious cycle could arise, when the lower value of government securities lowers bank assets, and raises the implicit financial guarantee, which in turn lowers government assets further. This means that the implicit guarantee is higher than what is shown above.

**Corporate Sector Balance Sheet**

| Assets | | Liabilities | |
|---|---|---|---|
| Corporate Assets | 120 | Loans ( Default-free value = 90, minus implicit loan guarantee – or put option – of 2.8) | 87.2 (43.6 loans from banks, and 43.6 to non-banks) |
| | | Corporate Equity | 32.8 |
| Total | 120 | | 120 |

**Banking Sector Balance Sheet**

| Assets | | Liabilities | |
|---|---|---|---|
| Loans (43.6) and Govt. Securities (30) | 73.6 ( -16.4) | Deposits | 81.3 |
| Financial Guarantee | 13.58 (+ 6.18) | Equity | 6.1 ( -7.2) |
| Total | 94.6 | | 94.6 |

**Public Sector Balance Sheet**

| Assets | | Liabilities | |
|---|---|---|---|
| Domestic assets | 90 (- 10) | Financial Guarantee to Banks | 13.58 (+ 6.18) |
| Foreign reserves | 30 (- 10) | Foreign Debt | 76.8 ( - 5.35) |
| | | Local-currency Debt & Other Liabilities | 29.9 ( - 20.55 ) |
| Total | 120 (- 20) | | 120 (- 20) |

(Units are in Billions of $)

## Annex 5 (cont.) - Figure A5 - 6

This scenario describes a negative shock to corporate sector assets (as in A5-2. above), in the case where the pension system contains one-half of corporate sector equity (in a defined benefit plan which also has a government guarantee). The results are shown below in Figure A5-6. A decline in corporate assets by $40 billion (from $120 to $80 billion) would cause the corporate equity value to drop by $26.9 billion to $5.9 billion (as compared with the base case). This increases the government guarantee to the pension system by $9 billion and the implicit guarantee to banks by $5.7 billion. In total, the government guarantees to pension system and banking system would increase to $22.3 billion, significantly higher than $7.4 billion in the base case).

## Figure A5 - 6

**Corporate Sector Balance Sheet**

| Assets | | Liabilities | |
|---|---|---|---|
| Corporate Assets | 80<br>(- 40) | Loans ( Default-free value = 90, minus implicit loan guarantee – or put option of 15.8) | 74.1<br>(- 13.1) |
| | | Corporate Equity | 5.9<br>(- 26.9) |
| Total | 80<br>(- 40) | | 80<br>(- 40) |

**Banking Sector Balance Sheet**

| Assets | | Liabilities | |
|---|---|---|---|
| Loans (90 -15.8) | 74.1<br>(- 13.1) | Deposits | 81.3 |
| Financial Guarantee | 13.3<br>(+ 5.7) | Equity | 6.1<br>(-7.2) |
| Total | 87.4<br>(- 7.2) | | 87.4<br>(- 7.2) |

**Pension System**

| Assets | | Liabilities | |
|---|---|---|---|
| Corporate Equity (initially 16.4) | 3<br>(- 13.4) | Defined Benefit (Present Value) | 12 |
| Financial Guarantee | 9<br>(+ 9) | | |
| Total | 12 | | 12 |

**Public Sector Balance Sheet**

| Assets | | Liabilities | |
|---|---|---|---|
| Domestic assets | 100 | Financial Guarantee to Banks & Financial Guarantee to Pension System | 13.3<br>+ 9<br>=22.3<br>(+ 14.7) |
| Foreign reserves | 40 | Foreign Debt | 78.9<br>(- 3.25) |
| | | Local-currency Debt & Other Liabilities | 39<br>(- 11.45) |
| Total | 140 | | 140 |

(Units are $ Billions.)

## Annex 6 - Application to Analysis of Debt Sustainability and Valuation of Sovereign Foreign-currency Debt

The application of CCA to the public sector leads to a new way to evaluate sovereign debt sustainability and a new way of valuing sovereign debt. Debt sustainability, as defined by macroeconomists, is a situation in which a sovereign borrower is expected to continue servicing its debt obligations without default or restructuring of the debt and without "unrealistically large" adjustments to the internal and external balance of payments. Sovereign debt sustainability analysis has relied on a "flow analysis and accounting ratio" approach. In most analyses of this type, the fiscal surplus is related to the interest rate paid on debt, GDP growth, and the current debt to GDP ratio.[34] The determination of debt sustainability depends on under what circumstances the debt to GDP ratio stabilizes.

By using financial tools and CCA analysis, a new perspective on debt sustainability can be developed. In the macro finance framework, one way to think about sustainability is when *spreads* on debt remain below a certain threshold level. This is the level at which the sovereign can continue servicing its debt obligations without default or restructuring of the debt and without "unrealistically large" adjustments in prices or internal and external balance of payments. The sovereign spread on foreign currency debt (associated with default free debt value, i.e. default barrier $DB_F$ ) is given by:

$$\text{Spread} = -\ 1/T \ln[\ N(d_2)\ +\ (A/\ (DB_F\ e^{-rt}))\ N(-d_1)]$$

The total asset (A) of the government and central bank includes foreign currency reserves plus the fiscal asset (measured in foreign currency terms). The term $\sigma_A$ is the volatility of the sovereign asset, r is the risk free rate and T is the time horizon for calculating the spread. As the term ($A/\ DB_F$) declines and/or $\sigma_A$ increases, the spread increases in a *non-linear* way and eventually turns sharply higher. The cost of borrowing increases when the spread increases. Sovereigns can get into a "slippery slope" situation where increasing interest rates paid on debt accelerates the amount of debt needed to service past debt and new debt at the higher rates.

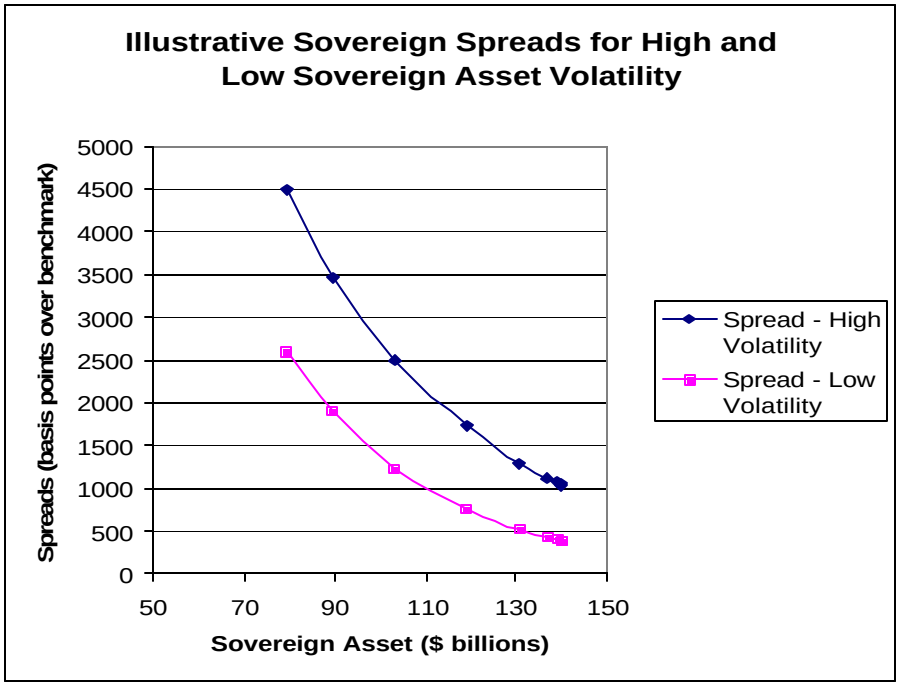
Uncertainty may increase the volatility of the fiscal asset (local currency volatility $_{LC}\sigma_{FA}$), because of uncertainties on the commitment of the authorities to maintain a prudent fiscal surplus/deficit. The fiscal asset may be anticipated to be lower in the future as well (due to lower taxes and/or higher expenditures). Also, depreciation and increased exchange rate volatility ($\sigma_{ER}$) may also occur. Higher $_{LC}\sigma_{FA}$ and higher $\sigma_{ER}$ can combine to produce a higher volatility of the fiscal asset $\sigma_{FA}$ [35] (measured in foreign currency terms). Thus, as shown in Figure A6-1, expectations of higher $\sigma_A$, and

[34] These relationships are usually primary fiscal surplus = (nominal interest rate paid on debt minus nominal GDP growth rate)/((1 plus GDP growth)(current debt to GDP))

[35] The relationships between local currency fiscal asset volatility, exchange rate volatility and correlation is: $(_{LC}\sigma^2_{FA} + \sigma^2_{ER} - 2\,\rho_{ER,FA}\,\sigma_{ER}\ _{LC}\sigma_{FA})^{1/2} = \sigma_{FA}$

lower A, could raise spreads sharply. If the sovereign needs to roll-over and finance the debt at higher rates the default barrier, DB, goes up and could raise spreads further.

## Figure A6-1




It is interesting to note that *short-term borrowing of foreign exchange* by the government from senior creditors can lead to an increase in assets (if the proceeds are used to increase reserves) but this type of borrowing also leads to a nearly the same increase in the DB, so A/DB doesn't increase much. In this case, spreads are not likely to decline very much, unless the volatility is reduced.

More work on these issues might usefully focus on a comparison of this CCA spread based approach with the traditional debt sustainability in diagnosing vulnerable situations. The CCA spread based approach opens up the potential for evaluating a whole set of policy options that could affect public sector asset level, default barrier, and volatility of the asset, as well as volatility of local currency fiscal asset and the exchange rate. The framework gives quantitative credit risk analysis and expands the menu of policy instruments and tools that can be used to try to reduce spreads (a potentially richer analysis than the traditional debt sustainability approach, which focuses almost exclusively on fiscal deficit adjustments as the policy instrument and accounting ratios of debt to GDP as the target).

## Annex 7 - Macrofinancial CCA Balance Sheets and Traditional Macroeconomic Flow of Funds

Something quite similar to the traditional macroeconomic income accounts can be derived from a specific, static case of the general macrofinancial contingent claim equations. If one assumes that the volatility of assets in the CCA formulas is zero and equity does not change from one period to the next, then the equations collapse into something very similar the flow-of-funds framework.[36] To see this result, let's take the change in CCA value for a representative sector with two categories of liabilities (junior claim modeled as a call option and debt modeled as default barrier minus a put option). For a representative sector with two categories of liabilities, J (most junior claim or equity) and debt, with DB defined as the default-free value of debt. "A" is asset value, R represents reserves.

$J_T = (A_0 + R_0)N(d_1) - DB_0\, N(d_2)$ $\quad$ $J_{T+1} = (A_1 + R_1)\, N(d_1) - DB_1 N(d_2)$

With the assumption that junior claim value does not change, $J_{T+1} = J_T$

$(A_1 + R_1)\ N(d_1) - DB_1 N(d_2) = (A_0 + R_0)\, N(d_1) - DB_0\, N(d_2)$

and with the assumption that volatility of assets is zero, $\sigma_A = 0$, then, $N(d_1) = N(d_2) = 1$, so

$A_1 + R_1 - DB_1 = A_0 + R_0 - DB_0$

Note that with the assumption of $\sigma_A = 0$, then $N(-d_1) = N(-d_2) = 0$
Which means that the equation for the market value of debt at time T = $DB_0 - [\ DB\, e^{-r\tau}N(-d_2) - A_0\ N(-d_1)\ ]$, becomes $DB_0$, and $DB_1$ at time T+1. In other words, the value of debt now becomes the default-free value since the put option value in the debt has become zero. Rearranging the equation above results in:

$A_1 - A_0 = DB_1 - DB_0 + R_0 - R_1$

The left hand term is the change in "net asset" in period t+1 minus the asset in period t. This is the income minus consumption minus investment over the period, or Y-C-I.

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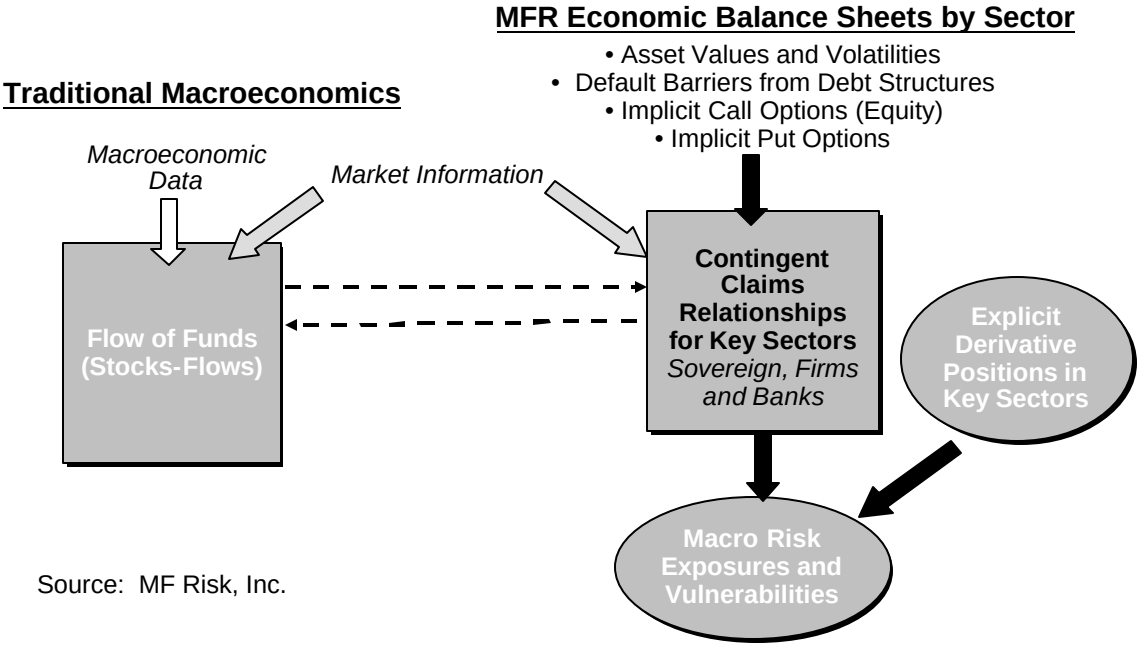
[36] The value of senior and junior claims are contingent claims that can be modeled as a derivative, F, dependent on the level of the asset, A, at time t in the future. If this asset changes in a *deterministic way* then one could use standard calculus to determine the change in the value of the derivative over time $dF(A_t,t) = (\partial F/\partial A)\, dA_t + (\partial F/\partial t)dt$. But if assets change in a *random way*, not a deterministic way, then stochastic calculus should be used; the tools of standard calculus are not suitable. Under standard calculus the change in F is determined for a small change in A, i.e. $\Delta A$, and $(\Delta A)^2$ is assumed to be so small it is negligible. But if A is a *random variable*, we cannot assume that $(\Delta A)^2$ is so small that it can be ignored. Treating $(\Delta A)^2$ as if it were zero would be equivalent to equating the variance and volatility of the asset to zero. (Neftci, 2000)

The result is: $Y - C - I = \Delta DB - \Delta R$

Income minus consumption (or expenditure) minus investment equals change in book value of debt[37] and change in reserves. This is analogous to the traditional macroeconomic flow-of-funds relationships (such as that used by the IMF).

Macroeconomic flow-of-funds can be viewed as a special case of macrofinance relationships. Using risk management tools to analyze risk exposures arising from *both* implicit options and explicit derivatives together can greatly enhance risk and policy analysis. The combined accounts – income/flow, mark-to-market balance sheets, and risk exposure measures comprise the three important sets of interrelated accounts for the economy. The traditional macroeconomic flow framework can be linked to the macro financial sectoral balance sheets and risk exposures (see Figure A7-1).

## Figure A7-1




Source: MF Risk, Inc.

[37] The change in default-free value of debt should be close to or equal to the change in book value of debt.

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