Counterparty Credit Risk

Modeling Counterparty Credit Exposure

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Discussion Plan

- **Modeling exposure at the trade level**
- **Scenario generation**
- **Trade valuation**
- **Counterparty-level exposure in the presence of netting agreements**
- **Modeling collateralized exposure**

Modeling exposure at the trade level

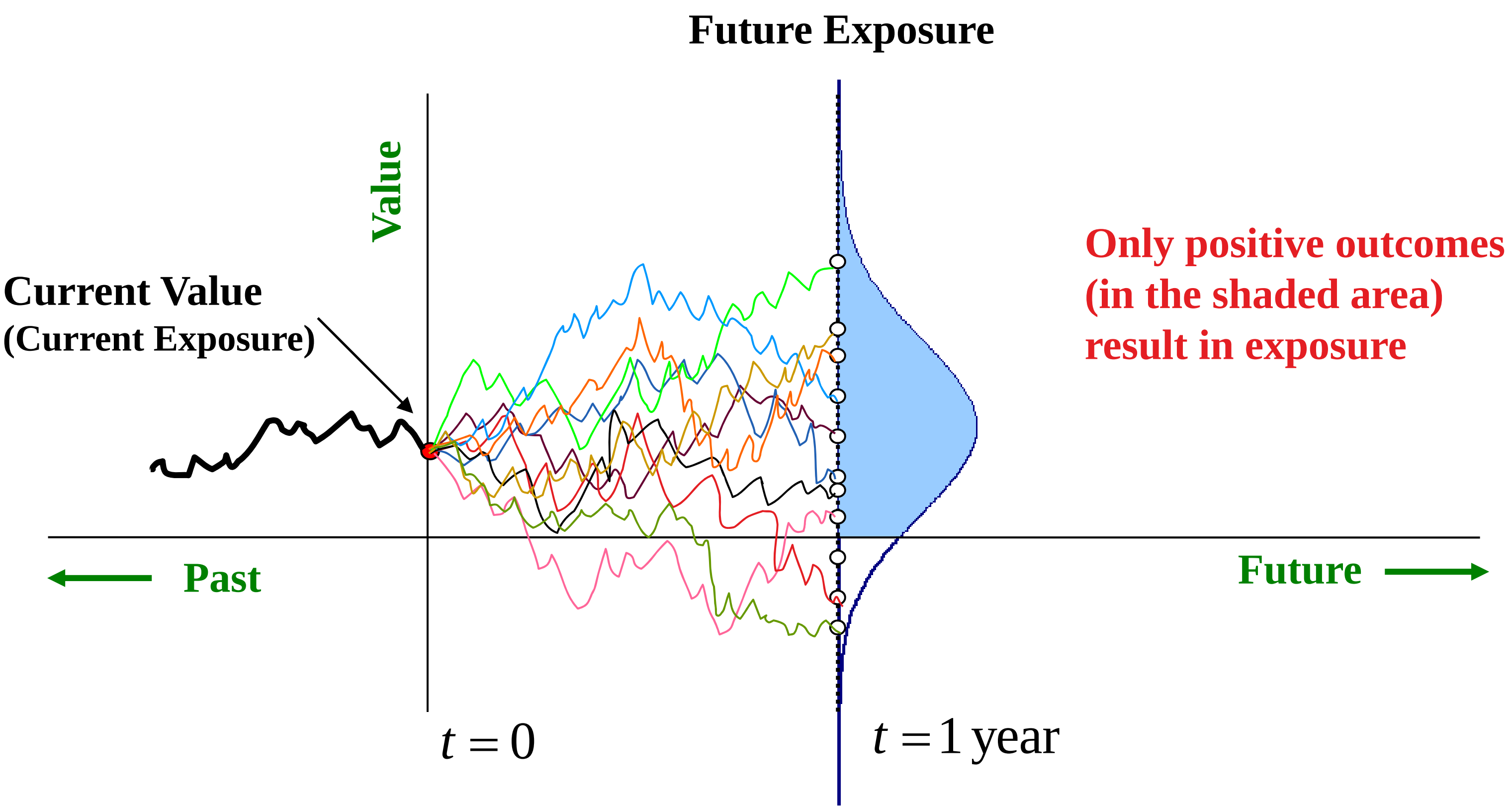
Stand-alone Exposure of a Trade

- Let us consider a counterparty with a ***single*** derivative contract i.
- The contract's market value is known only for current date $t = 0$. For any future date t, this value $V_i(t)$ is ***uncertain***.
- If the counterparty ***defaults*** at time τ prior to contract maturity, the trade is ***closed out*** at the current market value $V_i(\tau)$ and is likely to be ***replaced*** to restore the bank's market position
- The economic loss is equal to the ***replacement cost*** of the contract
  - If $V_i(\tau) > 0$, bank does not receive anything from defaulting counterparty, but has to pay $V_i(\tau)$ to another counterparty to replace the contract.
  - If $V_i(\tau) < 0$, bank receives $|V_i(\tau)|$ from another counterparty when it replaces, but has to forward this amount to the defaulting counterparty.
- Therefore, the ***stand-alone exposure*** $E_i(t)$ of contract i at time t is

$$E_i(t) = \max\{V_i(t), 0\}$$

Uncertainty of Future Credit Exposure

- Future market value and exposure are ***uncertain***!



Modeling Future Exposure

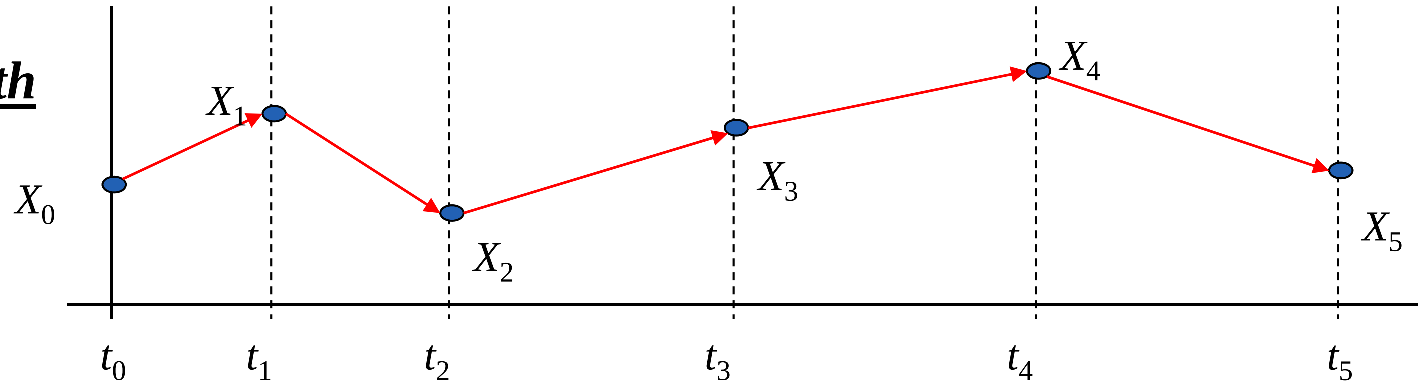
- At any future date *t* we should think of exposure as of a distribution of values.
- To obtain this distribution, we must be able to value the contract in the future.
- Contract value depends on the ***market risk factors*** affecting the derivative: FX rates, interest rates, stock prices, credit spreads…
- Therefore, modeling future exposure consists of two parts:
  - **Scenario generation:** simulating correlated risk factors in the future
  - **Trade valuation:** valuing trades in the future contingently on realization of risk factors
- Since counterparty can default at any time in the future, we need to simulate exposure for a large set (~100) of future time points to describe CCR adequately.

Scenario generation

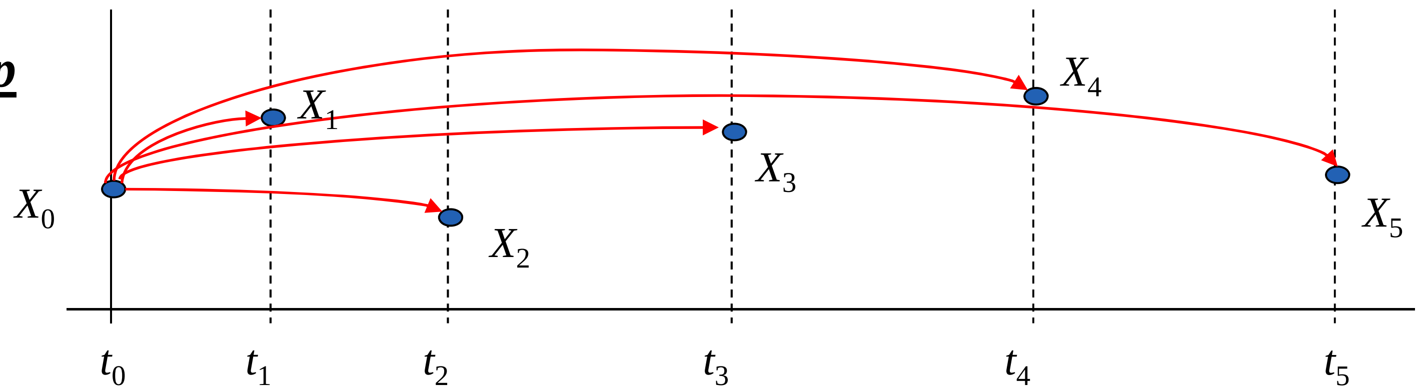
Two Approaches to Scenario Generation

- Evolution of any risk factor is modeled via ***stochastic process***

Path-by-Path



Direct Jump



Which Approach to Choose?

- Path by path simulation is a better approach because
  - More accurate valuation of ***path-dependent*** and ***American*** derivatives
  - If counterparty defaults are simulated, more accurate loss distribution
- Direct jump to simulation time point can be used because
  - Exposure at each simulation time point depends on the distribution of the trade values ***at*** that simulation time point
  - Risk factors are often modeled as Markov processes that have closed form solution
- Advantages of direct jump to simulation time point
  - Easier to implement
  - Different probability measures can be used at different simulation time points

Generalized Geometric Brownian Motion

- ***FX rates*** and ***equity prices*** are often modeled via ***generalized Geometric Brownian Motion***

  $$dX(t) = \mu(t)X(t)dt + \sigma(t)X(t)dw(t)$$

  where $\mu(t)$ and $\sigma(t)$ are instantaneous ***drift*** and ***volatility***, and $w(t)$ is a ***standard Brownian motion*** (aka a Wiener process)

- Stochastic differential equation (SDE) above has a closed-form solution that is used for simulations:

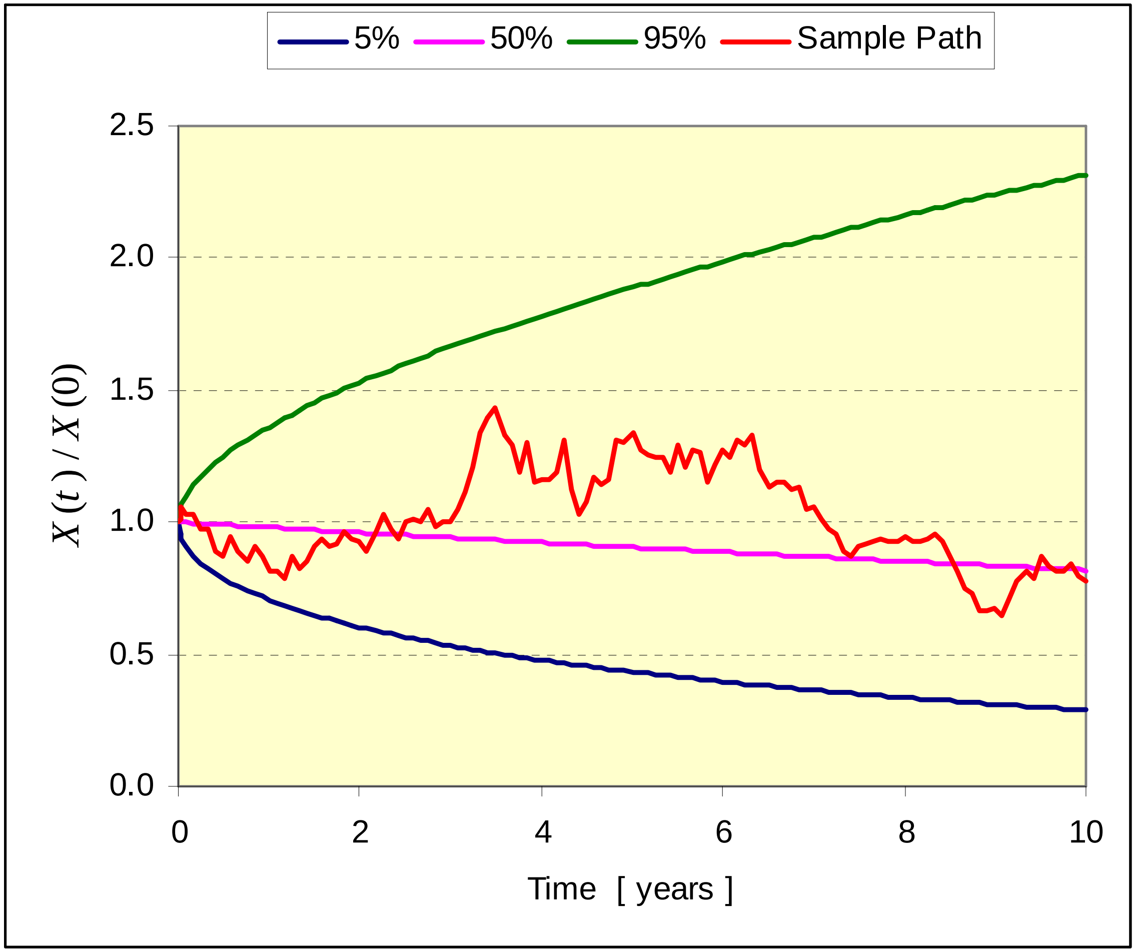
  $$X(t_k) = X(t_{k-1})\exp\left\{\alpha_k - \beta_k^2/2 + \beta_k\varepsilon_k\right\}$$

  where $\{\varepsilon_k\}$ are independent ***standard normal variables*** and

  $$\alpha_k = \int_{t_{k-1}}^{t_k} \mu(t)\,dt \qquad \beta_k^2 = \int_{t_{k-1}}^{t_k} \sigma^2(t)\,dt$$

Example of Geometric Brownian Motion

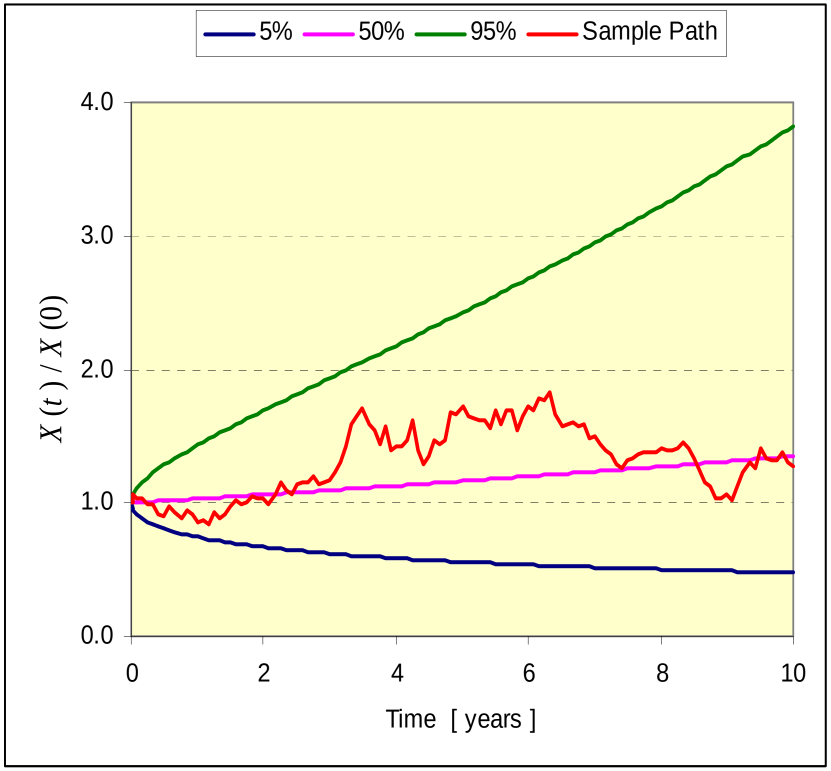
- **Assumptions**: $\mu(t) = 0$ and $\sigma(t) = 20\%$ for all t



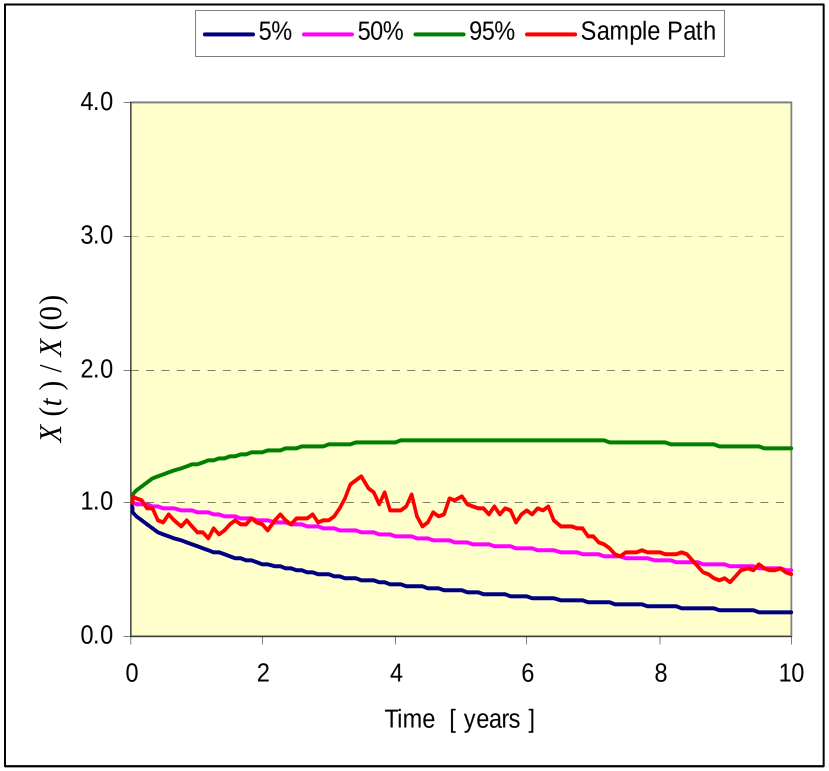
Effect of Drift

- Positive (negative) ***drift*** shifts all scenarios upwards (downwards)

$\mu(t) = 5\%$



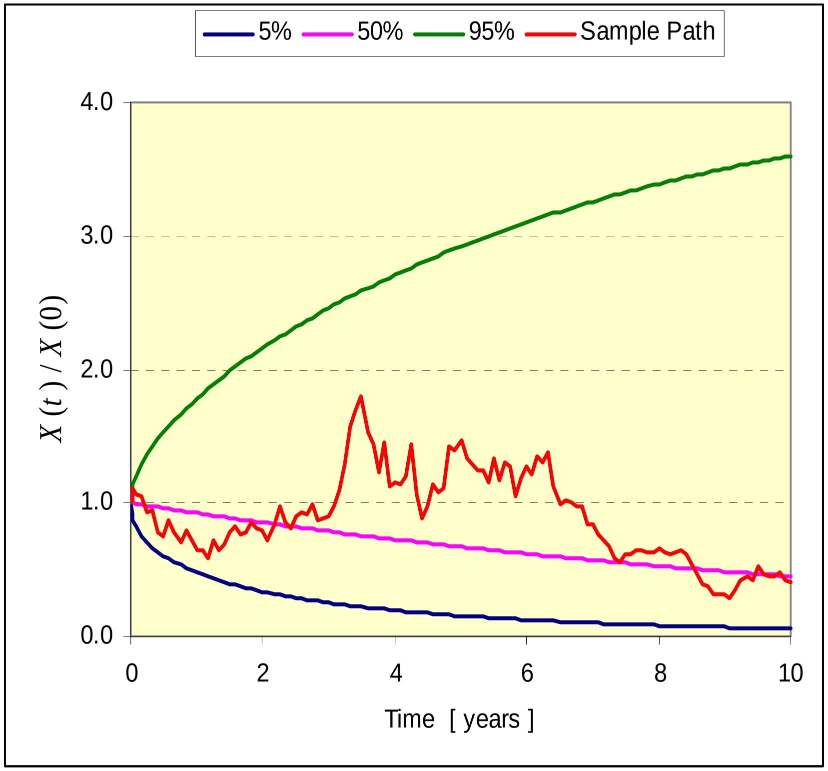
$\mu(t) = -5\%$



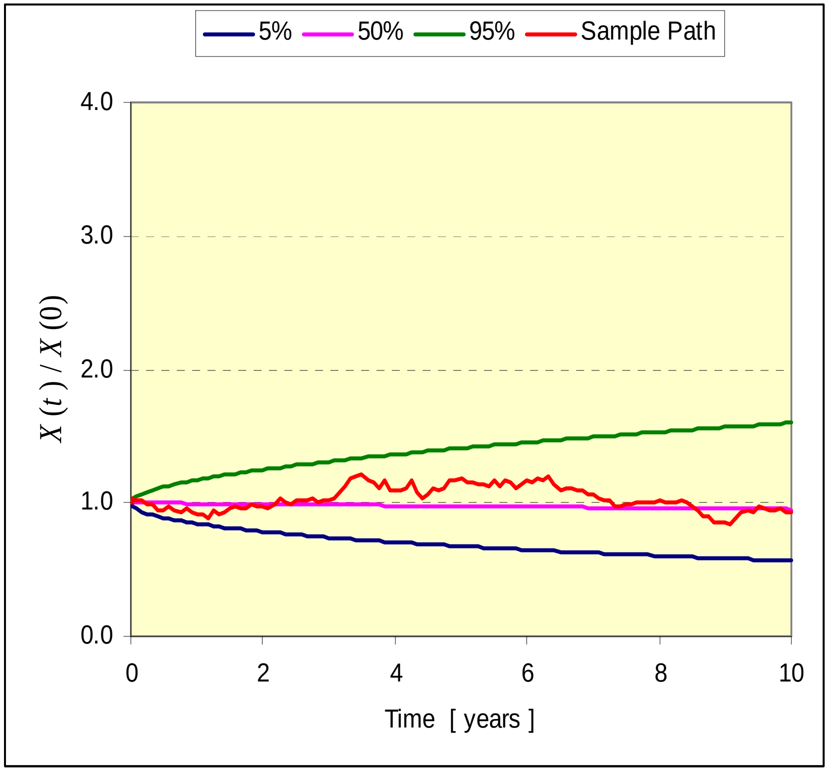
Effect of Volatility

- ***Volatility*** controls the width of the probability distribution

$$\sigma(t) = 40\%$$



$$\sigma(t) = 10\%$$



Geometric Mean Reverting Process

- ***Interest rate*** scenario models often include ***mean reversion***

- A version of ***Black-Karasinski process*** is often used:

$$dX(t) = -a\left(\ln X(t) - \ln \bar{X}\right) X(t)\,dt + \sigma(t) X(t)\,dw(t)$$

where a is ***mean-reversion rate*** and $\bar{X}$ is ***mean-reversion level***

- Solution to Black-Karasinski SDE:

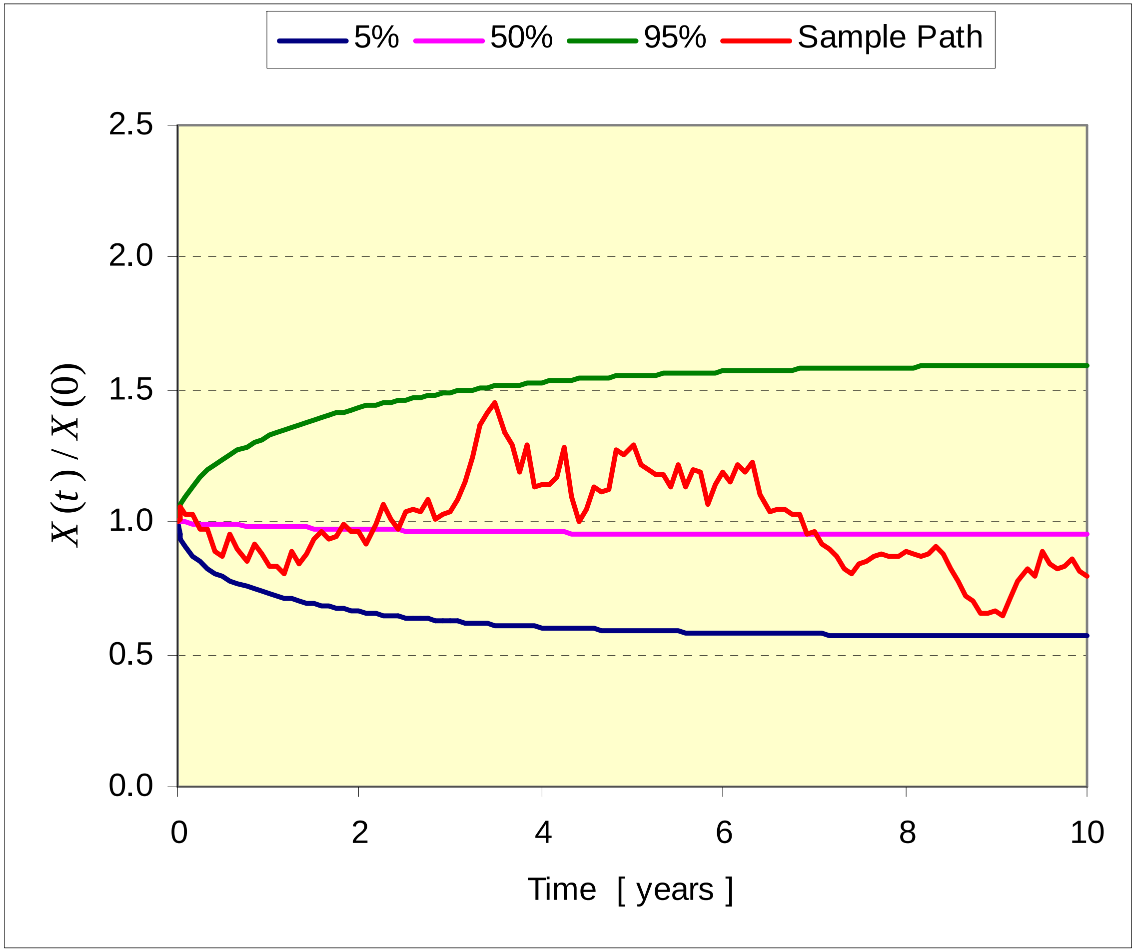
$$X(t_k) = \exp\left\{ e^{-a\Delta t_k} \ln[X(t_{k-1})] + (1 - e^{-a\Delta t_k}) \ln[\bar{X}] - \beta_k^2/2 + \beta_k \varepsilon_k \right\}$$

where

$$\beta_k^2 = \int_{t_{k-1}}^{t_k} \sigma^2(t) e^{-2a(t_k - t)}\,dt$$

Example of Black-Karasinski Process

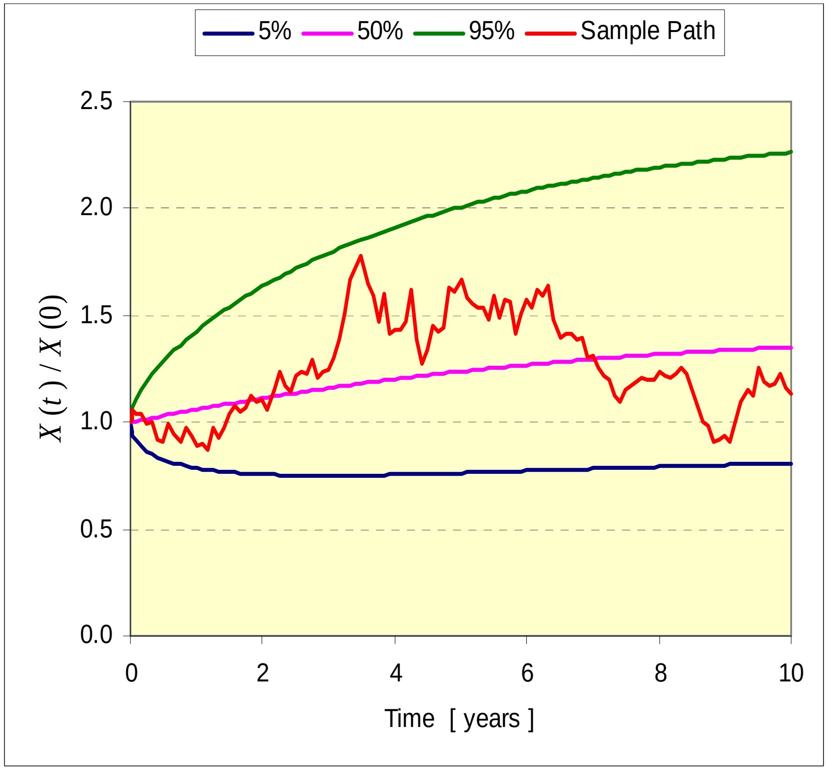
- **Assumptions**: $\sigma(t) = 20\%$, $\bar{X} = X(0)$ and $a = 20\%$ for all t

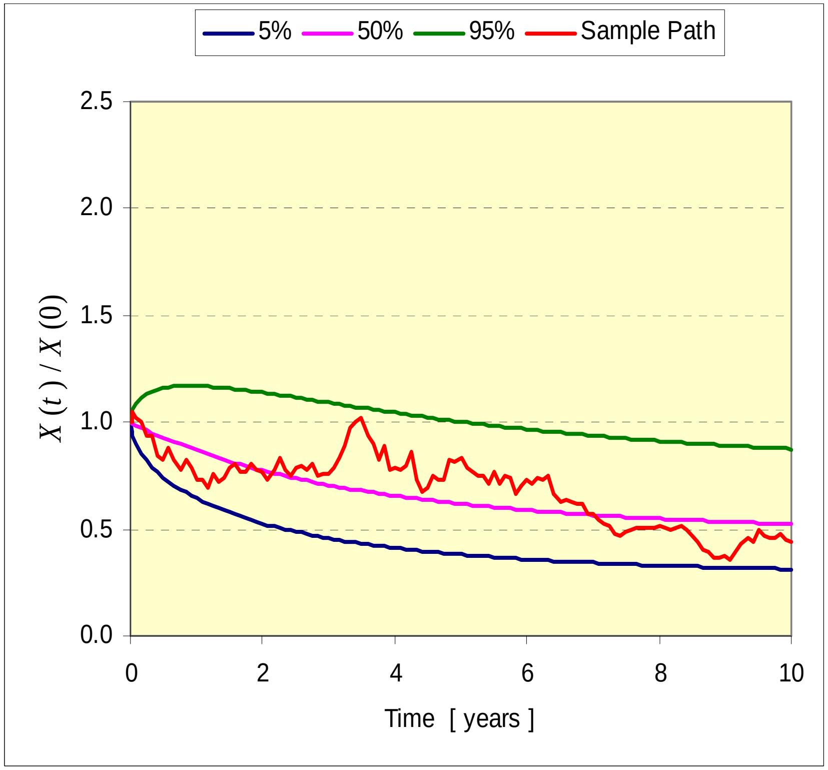


Effect of Mean Reversion Level

- All scenarios are pulled towards the ***mean reversion level (MRL)***. The further away from the MRL is scenario, the stronger the pull.

$$\bar{X} = 1.5 \cdot X(0) \qquad \bar{X} = 0.5 \cdot X(0)$$

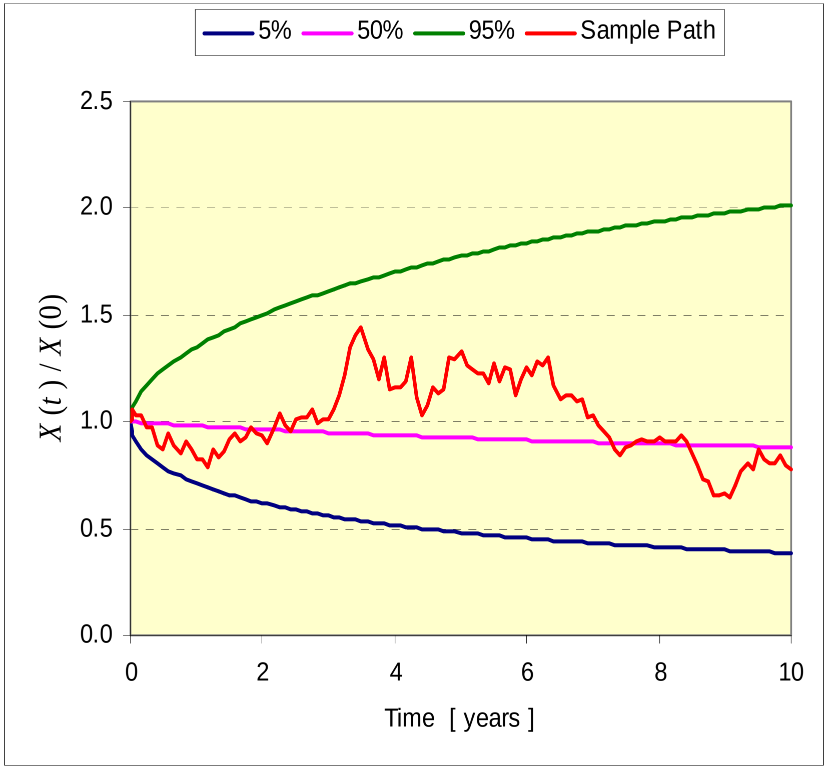




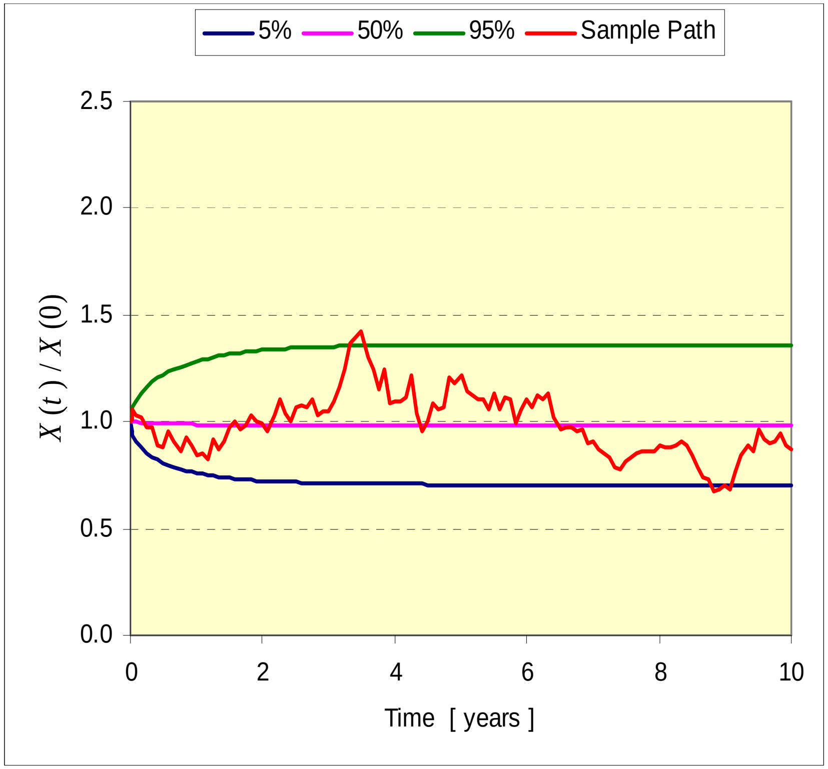
Effect of Mean Reversion Rate

- ***Mean reversion rate*** determines the strength of the pull towards the ***mean reversion level***

$a = 5\%$



$a = 50\%$



Trade valuation

Valuation at Future Scenarios

- ***Risk management valuation*** conditional on future scenarios radically differs from ***front office valuation***
  - Requirements on computation time differ by orders of magnitude!
- Risk management valuation must be done
  - for each instrument in the entire portfolio
  - for a few thousand risk factor scenarios
  - at ~ 100 simulation time points
- Therefore, such time-consuming valuation methods as Monte Carlo simulations are not acceptable
  - Simpler models that allow for closed-form solutions and/or relatively accurate analytical approximations are typically used

Example: European Option

- For ***European options*** on a single underlying (e.g. FX rate, equity) ***Black-Sholes*** formula with implied volatility is often used

$$V_{\mathrm{EO}}(t) = \varphi \cdot \left(e^{-q(T-t)} S(t)\,\mathrm{N}[\varphi d_1(t)] - B(t,T) K\,\mathrm{N}[\varphi d_1(t)] \right)$$

where

$$d_{1,2}(t) = \frac{\ln\left\{ S(t) e^{-q(T-t)} / [KB(t,T)] \right\} \pm (1/2)\sigma^2(T-t, S(t)/K)(T-t)}{\sigma(T-t, S(t)/K)\sqrt{T-t}}$$

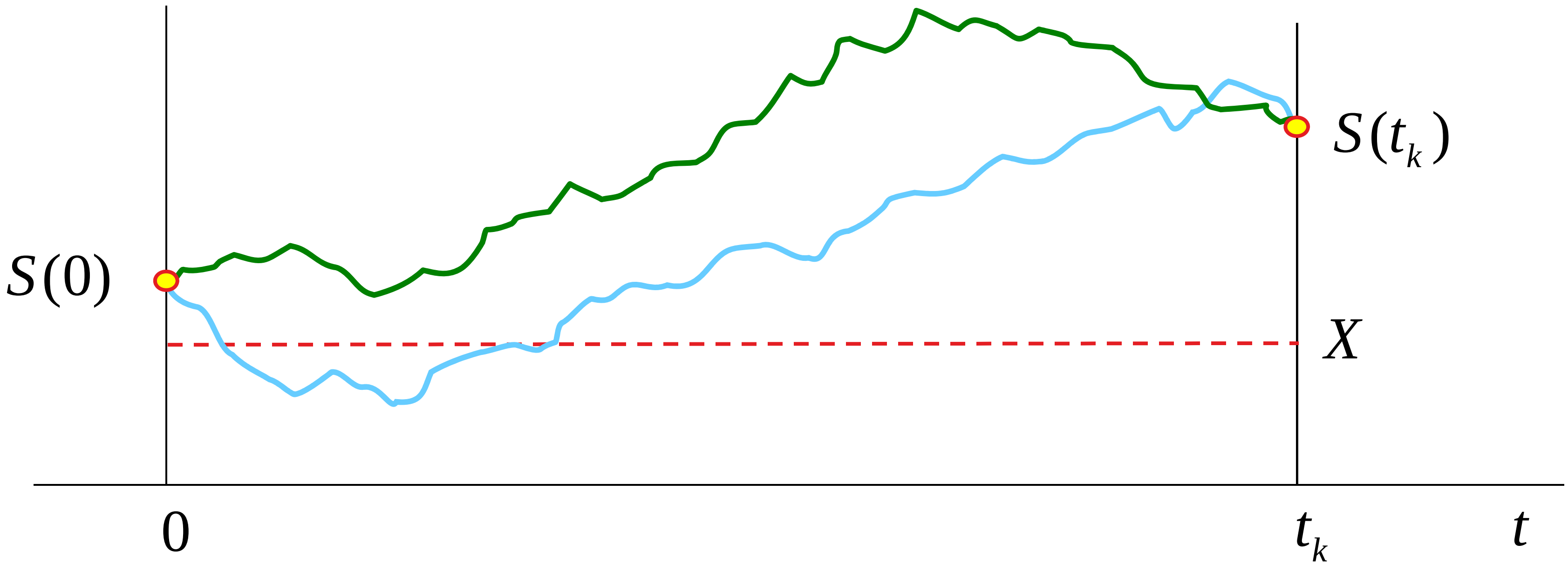
- Note that:
  - $S(t)$ and $B(t,T)$ are the ***price of the underlying*** and the ***discount factor***, respectively, ***simulated*** for time point t .
  - ***implied volatility*** $\sigma(T-t, S(t)/K)$ is calibrated to the option prices available in the market today

Valuation of Path-Dependent Derivatives

- For ***path-dependent*** and ***early exercise*** derivatives there is another reason why front office models cannot be used directly
- Scenarios are simulated for a discrete set of time points, while derivative value may depend on
  - the full continuous path to the simulation time point (e.g., barrier options)
  - a discrete set of dates that differs from the fixed set of simulation time points (e.g., Bermudan swaptions)
- Therefore, the ***incomplete*** information about the past precludes using front office models, which rely on ***full*** past information
- **Lomibao and Zhu (2005)** suggested conditional valuation
  - Path-dependent derivative is valued as expectation of its MTM value ***conditional*** on all available information from today to the simulation date

Example: Down-and-Out Barrier Option

- The simulated price $S(t_k)$ of the underlying at t_k is ***not sufficient*** to determine whether the option is alive or dead at t_k



- Under ***conditional valuation***, the value of the option at t_k is determined as the expectation of option's MTM value over all possible paths of S

$$V(t_k) = V_{\text{DOBO}}(t_k) \cdot \Pr\left[\min_{0 < t \le t_k} \{S(t)\} > X \;\middle|\; S(t_k) \right]$$

Counterparty-level exposure
in the presence of netting agreements

The Case of No Netting

- Very often a bank has multiple trades with a counterparty
  – Some of the trades may be fully or partially offsetting others
- From a legal point of view, each trade must be settled ***separately*** in the event of the counterparty's default
  – ***unless*** there are legally enforceable agreements that say otherwise
- Settling each trade separately means that the bank's exposure to the counterparty $E_{\text{cpt}}(t)$ is equal to the sum of stand-alone trade exposures

$$E_{\text{cpt}}(t) = \sum_i E_i(t) = \sum_i \max\{V_i(t), 0\}$$

- Trades with ***negative*** values cannot offset ones with ***positive*** value!

Example with No Netting

- Suppose that the portfolio consists of two opposite trades
  - When both counterparties want to get out of a trade they often enter into the opposite trade (unwinding) instead of cancelling the original one.
- While the portfolio value is identically zero, exposure is not!
- If there are only two states of the world, and the trades can assume values +$10 and -$10, we have

	Market Value			Exposure (no netting)		
	Trade 1	Trade 2	Total	Trade 1	Trade 2	Total
Scenario 1	10	-10	**0**	10	0	**10**
Scenario 2	-10	10	**0**	0	10	**10**

- When the counterparty defaults, the bank
  - forwards $10 to the counterparty to settle the trade with negative value
  - receives nothing from the counterparty when settling the "positive" trade

Netting Agreements

- ***Netting agreement*** is a legally binding contract between two counterparties that, in the event of default of one of them, allows aggregation of transactions before settling claims.
  - Derivatives with positive value at the time of default offset the ones with negative value; only the net value needs to be paid.
- Suppose that there is a netting agreement between the bank and the counterparty that covers the entire portfolio.
- Then, the bank's exposure to the counterparty is

$$E_{\text{cpt}}(t) = \max\left\{\sum_i V_i(t),\ 0\right\}$$

- If the example of two opposite trades included a netting agreement, the exposure would be ***zero***.

Multiple Netting Agreements

- From a risk management point of view, a single netting agreement covering all the trades would be an ideal solution
- However, because of operational constraints or other factors, counterparties often use multiple netting agreements that cover non-overlapping subsets of the portfolio
  - There may also be trades that are not covered by any netting agreement
- **Netting set**: a set of trades under a single netting agreement or a single non-nettable trade
- Counterparty-level exposure is given by

$$E_{\text{cpt}}(t) = \sum_k E_{\text{NS}_k}(t) = \sum_k \max\left\{ \sum_{i \in \text{NS}_k} V_i(t),\, 0 \right\}$$

where NS_k denotes the netting set number k.

Aggregation

- We have seen that to model ***trade-level*** exposure, one needs to complete two steps: ***scenario generation*** and ***trade valuation***
- To arrive at the distribution of the ***counterparty-level*** exposure, another step is necessary: ***aggregation***.
- Aggregation process is used to calculate the counterparty-level exposure at the scenario level
  - Obtain trade values calculated for a given scenario
  - Determine netting sets from netting agreement information
  - For each netting set, calculate portfolio value
    - If there is a margin agreement, calculate collateral and subtract it from the portfolio value
  - Obtain netting-set-level exposure by applying zero floor to portfolio value
  - Obtain counterparty-level exposure by adding netting-set-level exposures

Modeling collateralized exposure

Collateralization

- One of the most popular and effective techniques of mitigating and controlling CCR is ***collateralization***
- ***Similar*** to the case of ***lending risk***, collateral is posted to reduce credit exposure
- ***Unlike*** the case of ***lending risk***, collateral management is very complex:
  - **Uncertainty of credit exposure:** Because credit exposure changes unpredictably daily, it is not sufficient to post collateral once – collateral must be posted and returned frequently
  - **Bilateral nature of CCR:** If both counterparties want to limit their exposure, then either counterparty is required to post collateral when the other's exposure rises
- The rules of posting collateral are specified in a legally enforceable ***margin agreement*** signed by both counterparties

Features of Margin Agreements

- **Type & Currency of Collateral:** Cash has become the most common type of collateral.
- **Unilateral/Bilateral:** Margin agreement can be either ***unilateral*** (in one of the counterparties' favor) or ***bilateral***.
- **Threshold(s):** Counterparty has to post collateral if the unsecured exposure of the other counterparty exceeds ***threshold***.
- **Minimum Transfer Amount (MTA):** If the amount of collateral that needs to be posted or returned is less than the ***MTA***, no transfer of collateral occurs.
- **Margin Call Frequency:** This is the time period which specifies how often revaluation of the portfolio has to be performed to determine the amount of collateral (if any) that needs to be posted or returned.

Exposure with Margin Agreements

- Bank's exposure to the counterparty is

$$E_{\text{cpt}}(t) = \sum_{k} \max\left\{ \left(\sum_{i \in \text{NS}_k} V_i(t) \right) - C_k(t),\ 0 \right\}$$

where $C_k(t)$ is the market value of the collateral for netting set k at time t.

  - If NS_k is not covered by a margin agreement, then $C_k(t) \equiv 0$

- We have assumed the following sign convention:
  - $C_k(t) > 0$: at time t the bank holds collateral in the amount $|C_k(t)|$
  - $C_k(t) < 0$: at time t the counterparty holds collateral in the amount $|C_k(t)|$
  - $C_k(t) = 0$: at time t neither the bank nor the counterparty holds collateral

Unilateral Margin Agreement

- To simplify the notations, we will consider a single netting set:

$$E_{\text{cpt}}(t) = \max\left\{V(t) - C(t),\, 0\right\}$$

where $V(t)$ is the portfolio value for the netting set at time t :

$$V(t) = \sum_i V_i(t)$$

- Let's consider a ***unilateral*** margin agreement (in bank's favor) with threshold $H_{\text{cpt}} > 0$ and minimum transfer amount MTA.
- It is difficult to model collateral subject to MTA exactly because that would require daily simulation time points.
- In practice, the actual threshold H_{cpt} is often replaced with the effective threshold $H_{\text{cpt}}^{(\text{e})}$ defined as

$$H_{\text{cpt}}^{(\text{e})} = H_{\text{cpt}} + \text{MTA}$$

Naive Approach

- Collateral covers excess of portfolio value $V(t)$ over threshold $H_{\text{cpt}}^{(e)}$

$$C(t) = \max\left\{V(t) - H_{\text{cpt}}^{(e)}, 0\right\}$$

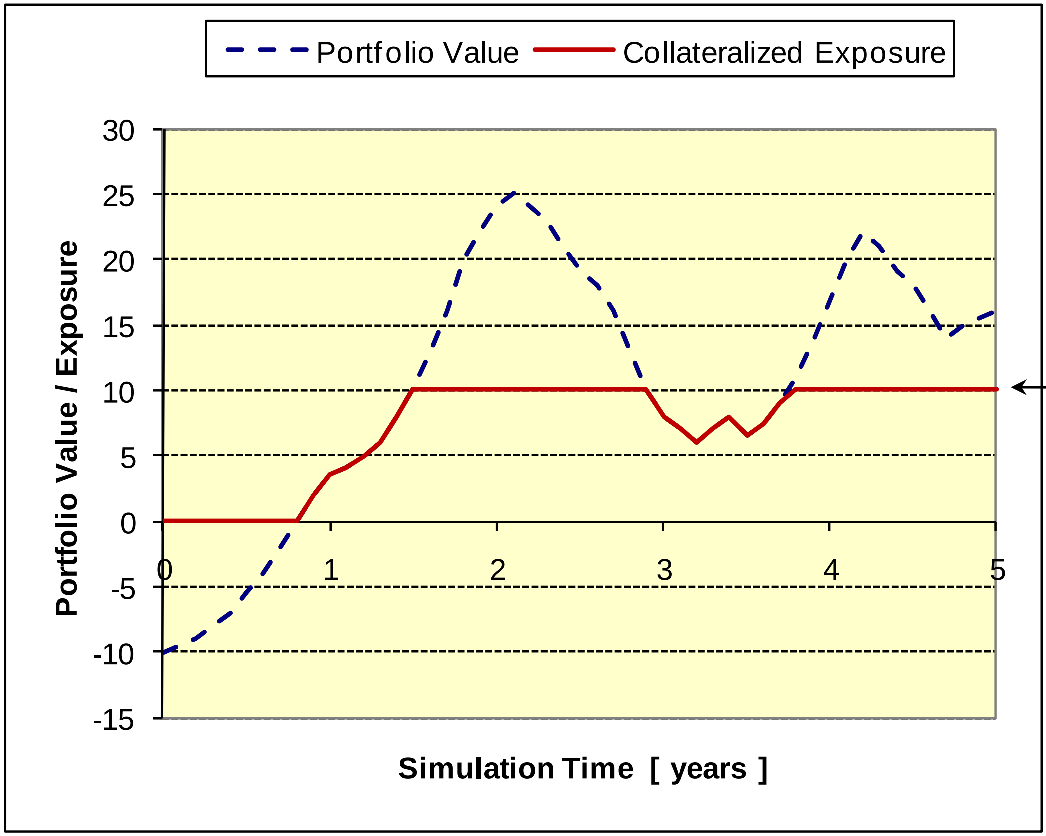
- Therefore, collateralized exposure is

$$E_{\text{cpt}}(t) = \max\left\{V(t) - C(t),\ 0\right\} = \min\left\{[V(t)]^{+}, H_{\text{cpt}}^{(e)}\right\}$$

- Thus, ***any scenario*** of collateralized exposure is limited by the ***threshold*** from above and by ***zero*** from below.
- This simple approach implicitly assumes that
  - collateral is delivered ***immediately***
  - the portfolio is settled and replaced ***immediately*** when collateral is not posted

Example of Exposure under Naive Approach

- Collateralized exposure for a sample scenario of portfolio value under the naive approach



$$H_{\text{cpt}}^{(\text{e})} = 10$$

Margin Period of Risk

- Even with daily margin call frequency, there is a significant delay δt , known as the ***margin period of risk (MPR)***, between a margin call that the counterparty does not respond to and the closeout/replacement of the portfolio if the counterparty defaults.
  - Margin calls can be ***disputed***, and it may take several days for the bank to realize that the counterparty is defaulting rather than disputing the call
  - There is a ***grace period*** after the bank issues notice of default. During this grace period the counterparty may still post collateral
  - It may take time to close out and replace ***complex trades***
- While δt is not known with certainty, it is usually assumed to be a fixed number, specified at the margin agreement level.
  - Assumed value of δt depends on margin call frequency and trade liquidity
  - Typical assumption for daily calls and liquid trades is $\delta t = 2$ weeks

Including MPR in the Model

- Suppose that at time $t-\delta t$ we have collateral $C(t-\delta t)$ and portfolio value is $V(t-\delta t)$
- Then, the amount $\Delta C(t)$ that should be posted by time t is

$$\Delta C(t) = \max\left\{V(t-\delta t) - C(t-\delta t) - H_{\text{cpt}}^{(\text{e})}, -C(t-\delta t)\right\}$$

  - Negative $\Delta C(t)$ means that collateral will be returned

- Collateral $C(t)$ available at time t is

$$C(t) = C(t-\delta t) + \Delta C(t) = \max\left\{V(t-\delta t) - H_{\text{cpt}}^{(\text{e})}, 0\right\}$$

- For comparison, collateral under the "naive" model is

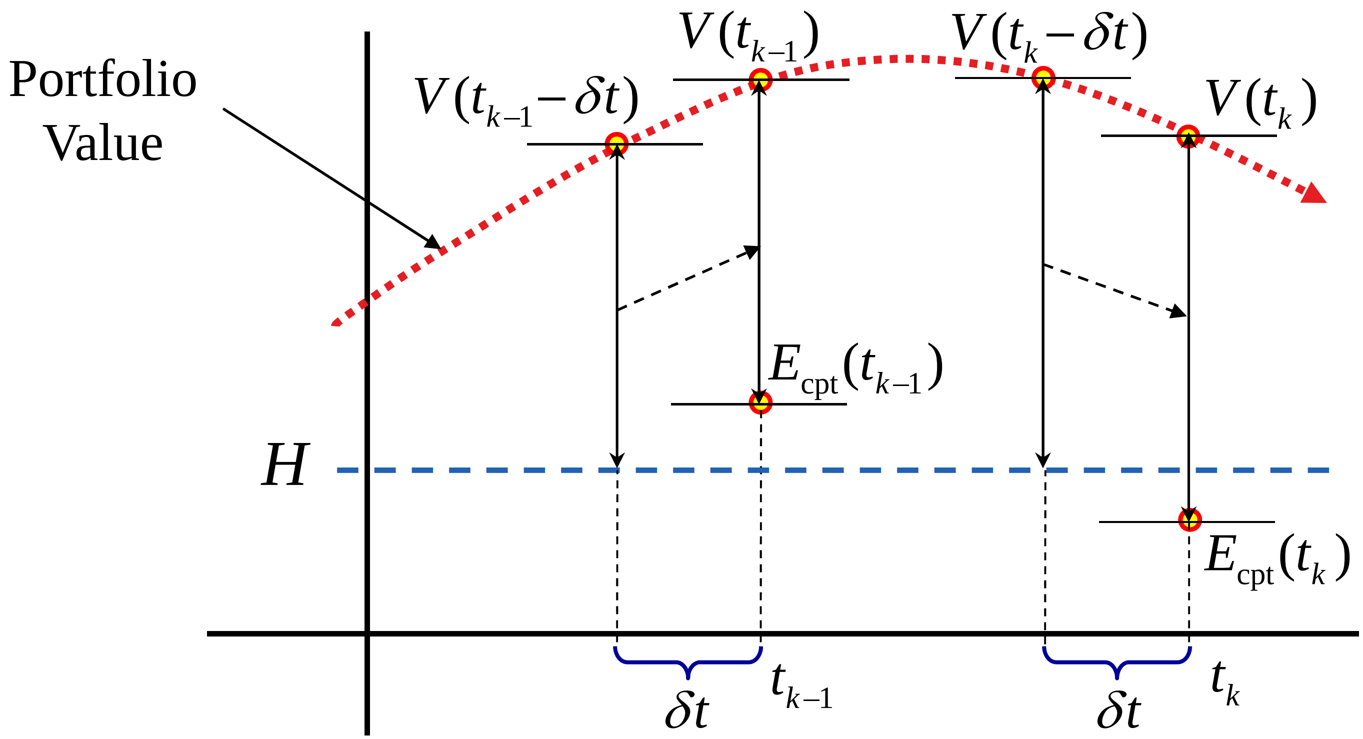
$$C_{\text{naive}}(t) = \max\left\{V(t) - H_{\text{cpt}}^{(\text{e})}, 0\right\}$$

- Thus, to determine collateralized exposure at time t, we need to simulate portfolio value both at $t-\delta t$ and at t.

Full Monte Carlo Method

- Under the ***delayed*** collateral delivery model, exposure is

$$E_{\text{cpt}}(t) = \min\left\{[V(t)]^+, [H_{\text{cpt}}^{(\text{e})} + V(t) - V(t-\delta t)]^+\right\}$$



Bilateral Margin Agreement

- Under a ***bilateral*** margin agreement, both the counterparty and the bank have to post collateral.
- Two thresholds are defined: $H_{\text{cpt}} \geq 0$ and $H_{\text{bnk}} \leq 0$
  - H_{bnk} is negative because we value trades from the bank's perspective
  - Bank posts collateral when portfolio value falls below H_{bnk}
  - Recall that we treat collateral posted by bank as a negative amount
- Two effective thresholds are specified:

$$H_{\text{cpt}}^{(\text{e})} = H_{\text{cpt}} + \text{MTA} \qquad H_{\text{bnk}}^{(\text{e})} = H_{\text{bnk}} - \text{MTA}$$

- After effective thresholds are defined, the bilateral margin agreement is treated as if it had zero MTA.

Collateral and Exposure for Bilateral MA

- Collateral available to bank at time *t* is given by

$$C(t) = \max\left\{V(t-\delta t) - H_{\text{cpt}}^{(\text{e})}, 0\right\} + \min\left\{V(t-\delta t) - H_{\text{bnk}}^{(\text{e})}, 0\right\}$$

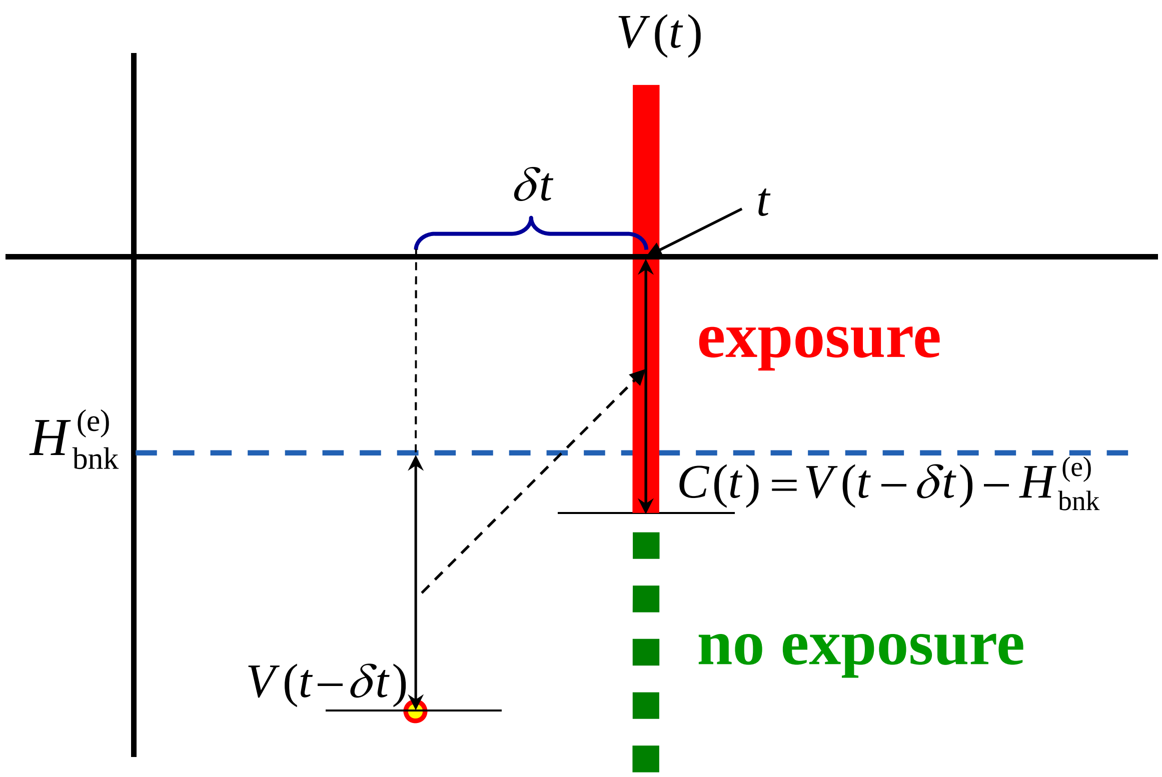
- The two terms above describe two types of future scenarios:
  - First term: the bank receives collateral $C(t) > 0$
  - Second term: the bank posts collateral $C(t) < 0$
- Note that both terms cannot be non-zero simultaneously!
- Bank's exposure to counterparty is still given by

$$E_{\text{cpt}}(t) = \max\left\{V(t) - C(t),\ 0\right\}$$

- If the counterparty defaults when the bank has posted collateral, is there any credit exposure for the bank?

Exposure from Posting Collateral

- When the bank posts collateral, it can experience loss if the portfolio value increases by more than $|H_{\text{bnk}}^{(e)}|$ over the MPR δt



$$E_{\text{cpt}}(t) = \max\{V(t) - C(t),\ 0\}$$

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