Counterparty Credit Risk

IAFE Credit Risk Event

March 24, 2009

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Citi Risk Architecture

Measuring the counterparty credit exposure profile

Counterparty credit exposure – single transaction

- **The potential exposure profile over time of a single OTC derivative is uncertain. It is contingent on the path market rates follow over time.**

Example 1: Forward FX, We buy GBP and sell US$ for settlement in two years at 1.5000 US$/GBP.

Random path of forward FX rate for a fixed settlement date, over life of forward transaction in **scenario 1**.

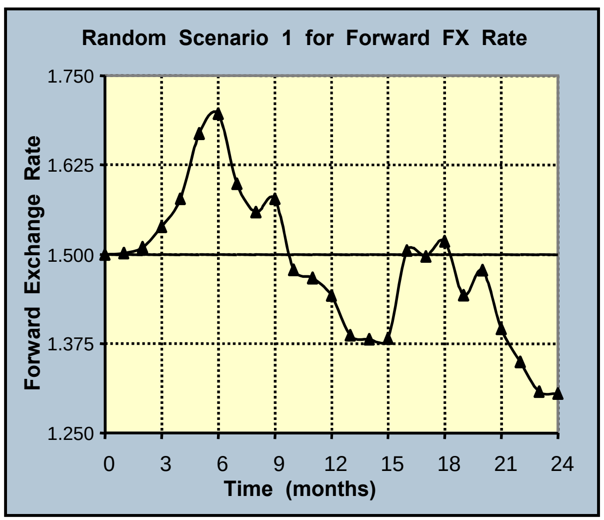
→

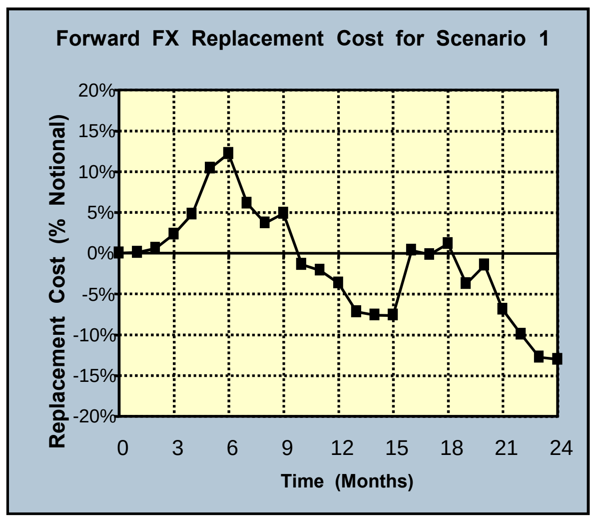
Profile of market value of forward FX transaction over its life, for scenario 1.

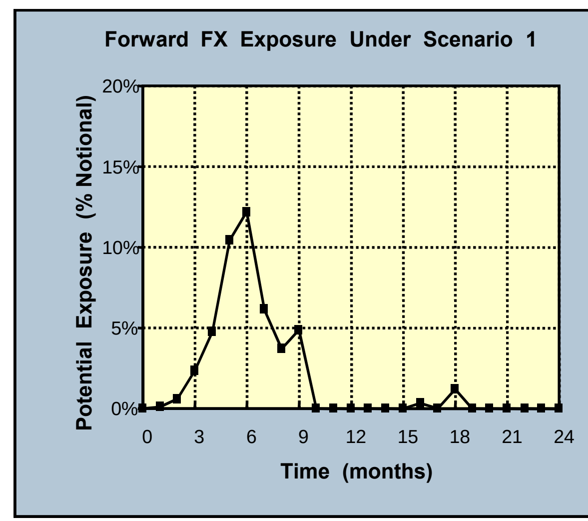
→

Exposure Profile of transaction for scenario 1.

We only have exposure when the contract has a positive value to us.

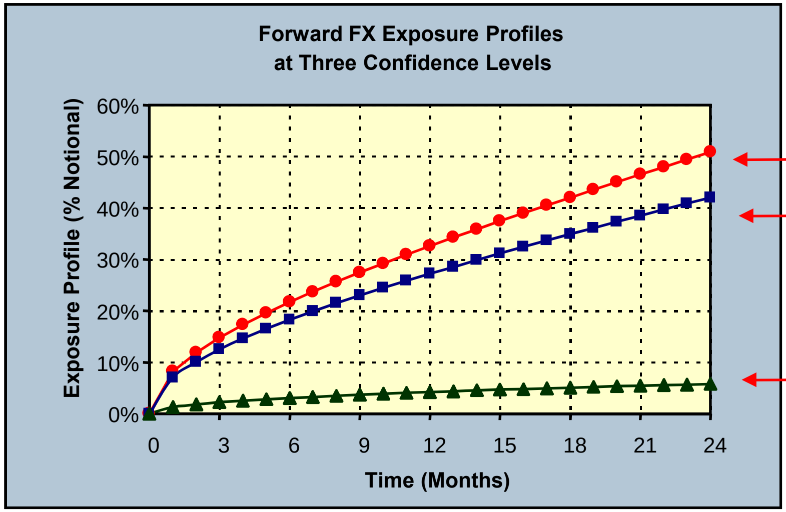






Potential Exposure Of A Single Transaction

Statistical Picture Of Potential Exposure



99.0% CL Profile.

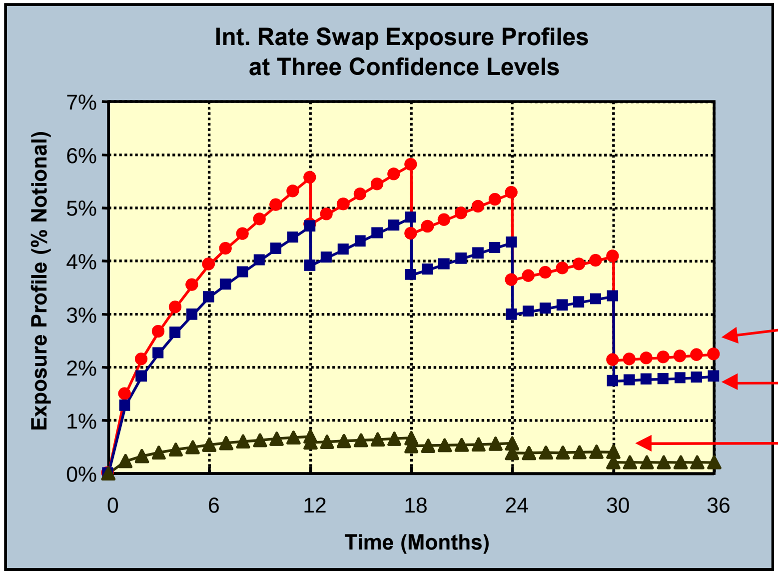
97.7% CL Profile.

EPE Profile

On the basis of thousands of such simulations we can represent the <u>potential exposure</u> over time <u>statistically</u>, at different confidence levels:

Three Exposure Profiles for a two year US$/GBP forward FX transaction. At <u>three confidence levels</u>:

- **99% CL** Exposure Profile
- **97.7% CL** Exposure Profile
- **Expected Positive** Exposure Profile



99.0% CL Profile.

97.7% CL Profile.

EPE Profile

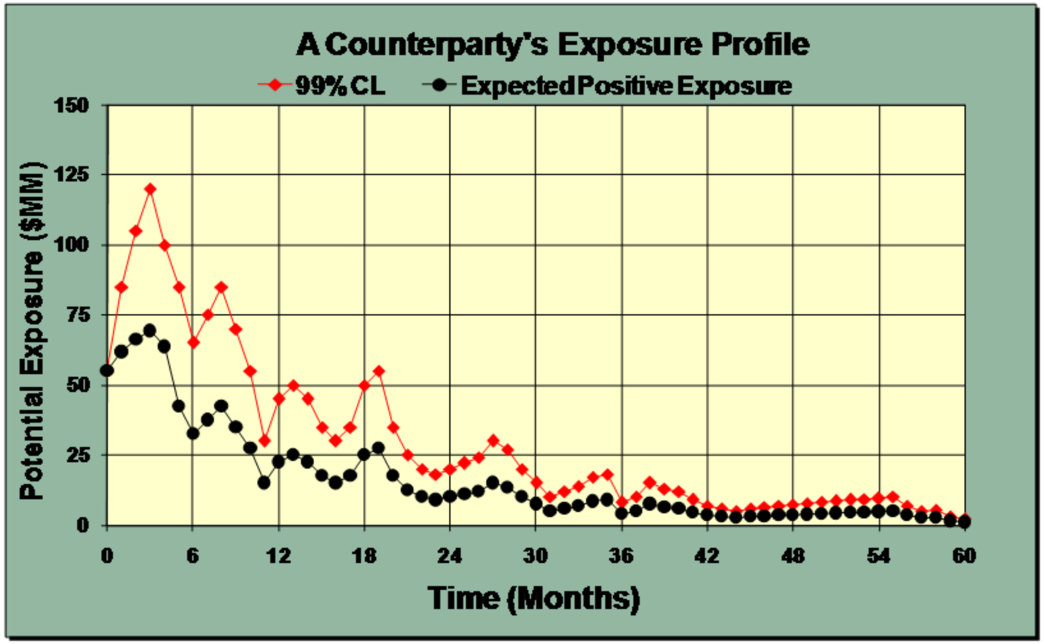
Three Exposure Profiles for a three year fixed/floating US$ interest rate swap. At <u>three confidence levels</u>:

- **99% CL** Exposure Profile
- **97.7% CL** Exposure Profile
- **Expected Positive** Exposure Profile

We also could have measured the potential liability at any confidence level

Exposure profile for a counterparty with many transactions

- **The <u>potential exposure profile</u> of a counterparty with many transactions is contingent on all the market factors that effect the value of those transactions as well as any legally enforceable risk mitigation agreements, such as netting or margin.**
- **It is complex to model because of the need to take into account the effect of legally enforceable risk mitigation agreements like netting and margin.**
- **If we simulate thousands of paths of the market we can represent the potential exposure profile of a portfolio of contracts at a specified confidence level:**



Potential exposure profile of a counterparty, at two confidence levels, over the lifetime of the transactions with the counterparty:

Assumes:

- **No additional transactions**
- **Contractual cash flows are set and settle over time.**
- **All legally enforceable risk mitigation agreements are taken into account**

We also could have measured the potential liability at any confidence level

Definition of EPE(t) and ENE(t) profiles

Simulated <u>positive exposure</u> of counterparty at time t, for <u>one potential path</u> of market rates over time:

$$\boldsymbol{E}^{+}(\boldsymbol{t}) = \sum_{m} \max\left(\left(\max\left(\sum_{j} \mathrm{PV}_{j,m}(\mathrm{t}), 0 \right) - \mathrm{CCVM}_{m}(\boldsymbol{t}) \right), 0 \right) \; + \; \sum_{k} \max\left(\sum_{j} \mathrm{PV}_{j,k}(\mathrm{t}), 0 \right) \; + \; \sum_{l} \max\left(\mathrm{PV}_{l}(\mathrm{t}), 0 \right)$$

Sum over netting/margin sets m *Sum over netting sets k* *Sum over unnetted transactions*

EPE(t) = Expected[E^{+}(t)] simulated over thousands of paths

Simulated <u>negative exposure</u> of counterparty at time t, for <u>one potential path</u> of market rates over time:

$$\boldsymbol{E}^{-}(\boldsymbol{t}) = \sum_{m} \max\left(\left(\min\left(\sum_{j} \mathrm{PV}_{j,m}(\mathrm{t}), 0 \right) - \mathrm{CCVM}_{m}(\boldsymbol{t}) \right), 0 \right) \; + \; \sum_{k} \min\left(\sum_{j} \mathrm{PV}_{j,k}(\mathrm{t}), 0 \right) \; + \; \sum_{l} \min\left(\mathrm{PV}_{l}(\mathrm{t}), 0 \right)$$

Sum over netting/margin sets m *Sum over netting sets k* *Sum over unnetted transactions*

ENE(t) = Expected[E^{-}(t)] simulated over thousands of paths

Why measure EPE or ENE?

- **Measurement of potential asset at different confidence levels**
  - **To calculate potential <u>credit exposure</u> at a high confidence level.**
  - **To calculate <u>potential margin</u> counterparty might have to post.**
  - **To calculate <u>Economic Capital due to default risk</u>.**
    - EC = Risk Weight(PD, LGD, Tenor) * EPE_one_year * α
  - **To calculate the <u>credit value adjustment</u> (CVA) for counterparty exposure.**
  - **To calculate the <u>potential asset that will have to be funded</u>.**

- **Measurement of potential liability at different confidence levels.**
  - **To measure <u>liquidity risk of own firm posting margin</u>.**
  - **To calculate the <u>bilateral CVA</u>.**
  - **To calculate the potential <u>liability that will fund other assets</u>.**

CVA

Economic Loss - Counterparty Risk - Economic Loss Analysis

- **Basic Question:**
  - **What should be the effect of credit spreads / risk rating on derivative valuation?**
  - **If all derivatives are (and should be) marked-to-market by discounting expected future cash flows at LIBOR bid/offer midpoint, then valuation would be:**
    - **Independent of counterparty's risk rating and**
    - **Independent of counterparty's credit spread.**
  - **If that were the case, changes in risk rating or spreads would not cause a change in economic value. Default only and economic loss analysis would be the same.**

Economic Loss - Counterparty Risk - Economic Loss Analysis

- **What is the credit risk premium for OTC derivatives?**

- **Let us first review the credit risk premium for a bond of a loan:**

  Bond/Loan Value = **PV of cash flows discounted at Risk Free Rate – Risk Premium$_{\text{loan/Bond}}$**

  = **PV of cash flows discounted at (Risk Free Rate + Spread)**

  Risk Premium $_{\text{Loan/bond}}$ $\cong$ **PV$_{\text{Risk Free}}$ * Duration * Spread$_{\text{Loan/bond}}$**

- **Let us try to apply the same logic to calculate the credit risk premium of a derivative:**

  Derivative Value = **PV of cash flows discounted at Risk Free Rate – Risk Premium**

- **However, the context for ascertaining the credit risk premium of derivatives is material different than it is for loans because of the different nature of their respective credit exposures. We need to:**
  - Perform a portfolio analysis of exposure
  - Take into account the uncertain future exposure
  - Evaluate significance, if any, of bilateral nature of exposure
    - In general, at any future date
      - Obligor could owe us (asset) or
      - We could owe obligor (liability)
    - Therefore how should we take the potential bilateral nature of exposure into account?

Economic Loss - Counterparty Risk - Economic Loss Analysis

Defining a unilateral and a bilateral CVA

- **Let us call the credit risk premium of the counterparty's portfolio its CVA.**

  CVA = Credit value adjustment for counterparty's credit risk

- **Therefore, the market value of derivative portfolio with a counterparty is:**

  = Σ MARKET VALUE (discounted at risk free rate) - CVA

Calculated on a portfolio basis, taking into account potential future exposure

- **Measuring the CVA of a counterparty: Modification of a proposal by Bollier & Sorensen.**

  Two perspectives on CVA: a Unilateral and a Bilateral perspective.

Credit premium of own firm's expected asset from derivatives with counterparty

Unilateral CVA: $CVA_{\text{COUNTERPARTY K, UNILATERAL}} = CVA^{+}_{\text{CNTPY K}}$

Bilateral CVA: $CVA_{\text{COUNTERPARTY K, BILATERAL}} = CVA^{+}_{\text{CNTPY K}} - CVA^{-}_{\text{CNTPY K}}$

Credit premium of own firm's expected liability from derivatives with counterparty.

Economic Loss - Counterparty Risk - Economic Loss Analysis

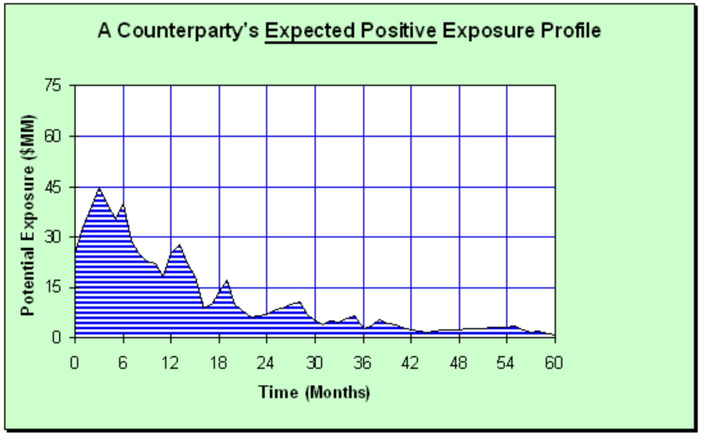
Defining a unilateral and a bilateral CVA

$$\text{Market Value}_{\text{CP Portfolio}} = \Sigma\, \text{MV}_{\text{CP Portfolio (risk free)}} - \text{CVA}_{\text{CP Portfolio}}$$

$$\text{CVA}_{\text{CP Portfolio\_Unilateral}} = \text{CVA}^{+}_{\text{CNTPY}}$$

$$\text{CVA}_{\text{CP Portfolio\_Bilateral}} = \text{CVA}^{+}_{\text{CNTPY}} - \text{CVA}^{-}_{\text{CNTPY}}$$

Expected amount CP will owe to own firm.

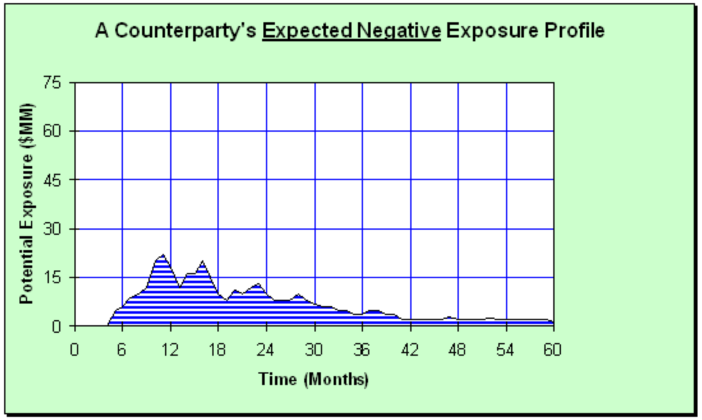


$\text{CVA}^{+}_{\text{CNTPY K}}$ (Credit premium of own's firm potential asset from CP K)

$$= \sum_{J} (\text{Expected Exposure}^{+}_{K,J} * \text{Forward CP Spread}_{K,J} * \Delta t_J * df_J)$$

Calculate and sum over each forward period J, for Counterparty K.

Expected amount own firm will owe to CP.



Should have also taken into account survival rate

$\text{CVA}^{-}_{\text{CNTPY K}}$ (Credit premium of CP K's potential asset from our own firm)

$$= \sum_{J} (\text{Expected Exposure}^{-}_{K,J} * \text{Forward Own Firm's Spread}_{J} * \Delta t_J * df_J)$$

Calculate and sum over each forward period J, for Counterparty K

Economic Loss - Counterparty Risk - Economic Loss Analysis

Implications of a unilateral and a bilateral CVA

$$\textbf{Market Value}_{\text{CP Portfolio}} = \Sigma\,\textbf{MV}_{\text{CP Portfolio (risk free)}} - \textbf{CVA}_{\text{CP Portfolio}}$$

$$\textbf{CVA}_{\text{CP Portfolio\_Bilateral}} = \textbf{CVA}^{+}_{\text{CNTPY}} - \textbf{CVA}^{-}_{\text{CNTPY}} \quad \text{(bilateral perspective)}$$

$$\textbf{CVA}_{\text{CP Portfolio\_Unilateral}} = \textbf{CVA}^{+}_{\text{CNTPY}} \quad \text{(unilateral perspective)}$$

<u>Examples</u>

Example 1:

- **Assume:**
  - **Only One Swap With Counterparty**
  - **Counterparty And Own Firm Have Same Risk Rating.**
  - **Potential Change in Value Has Symmetric Shape For Pay or Receive Fixed Swaps. (e.g. flat yield curve).**
- **Consequence for unilateral and bilateral CVA**

Example 2:

- **Assume:**
  - **Only One Swap With Counterparty**
  - **Counterparty Is BBB And Own Firm Is AA.**
- **Consequence for unilateral and bilateral CVA**

Two Perspectives on Counterparty Credit Risk

CVA

- **Market Value CP Portfolio = Σ $MV_{CP\ Portfolio\ (risk\ free)}$ - $CVA_{CP\ Portfolio}$**

- **Two choices:**
  - **Credit Risk Perspective**
    - **CVA is treated as part of credit risk. This bifurcation of counterparty risk into a) market risk component and b) credit risk component is similar to what occurs under accrual accounting, where the risk of a loan portfolio is bifurcated into a) accrual interest rate risk and b) credit risk.**
    - **From this perspective hedging counterparty risk is done by buying a CDS or a CCDS from a third party, in the same way that the credit risk of a loan is hedged by buying a CDS or a Guarantee from a third party**
  - **Full Market Risk Perspective:**
    - **CVA is treated as part of market risk, in the same way that the spread of a bond is treated as part of market risk.**
    - **Dynamically delta hedging CVA is equivalent to transforming counterparty credit risk to market risk and hedging it like market risk.**

Dynamically Hedging Counterparty Risk

- **Market Value CP Portfolio** $= \Sigma MV_{\text{CP Portfolio (risk free)}} - CVA_{\text{CP Portfolio}}$

Gives rise to market risk EC. *Gives rise to counterparty credit risk EC.*

$$CVA_{\text{Unilateral}} = \text{Spread} * \sum_{k=1}^{\text{Full life of portfolio}} EPE_k \ \Delta t_k \ df_k$$

- **For a specific portfolio of U.S. Dollar LIBOR interest rate swaps, the discounted area under the EPE curve, $\Sigma(EPE_k \, \Delta t_k \, df_k)$, will be a function of the terms and conditions of all the swaps and the term structure and volatility of the U.S. Dollar LIBOR yield curve.**

$$\begin{aligned} CVA &= \text{Spread} * \sum_{k=1}^{\text{Full life of portfolio}} EPE_k \ \Delta t_k \ df_k \\ &= \text{Spread} * g(\text{Contract T \& Cs, Risk Mitigants}; \{r(t)_k\}, \{\sigma_k\}) \\ &= \text{Spread} * g(\text{Contract T \& Cs, Risk Mitigants}; x_k, \sigma_k) \\ &= \text{Spread} * g(\text{Contract T \& Cs, Risk Mitigants}; x, \sigma) \end{aligned}$$

- *EPE of counterparty depends on Terms and conditions of contracts, legally enforceable risk mitigant agreements, yield curve and assumed volatilities and correlations.*
- *Represent interest rates by a single variable X and interest rate implied vol by a single variable σ.*

$$\Delta CVA = \Delta \text{Spread} * \sum_{k=1}^{\text{Full life of portfolio}} EPE_k \ \Delta t_k \ df_k + \text{Spread} * \frac{\partial CVA}{\partial x} * \Delta x + \text{Spread} * \frac{\delta CVA}{\delta \sigma} * \Delta \sigma + \frac{\partial^2 CVA}{\partial \text{Spread} \partial x} * \Delta \text{Spread} * \Delta x + \ldots$$

Dynamically Hedging Counterparty Risk

$$CVA = Spread * \sum_{k=1}^{\text{Full life of portfolio}} EPE_k \; \Delta t_k \; df_k$$

$$\Delta CVA = \underbrace{\Delta Spread * \sum_{k=1}^{\text{Full life of portfolio}} EPE_k * \Delta t_k * df_k}_{} + \underbrace{Spread * \frac{\partial CVA}{\partial x} * \Delta x + Spread * \frac{\delta CVA}{\delta \sigma} * \Delta \sigma}_{} + \underbrace{\frac{\partial^2 CVA}{\partial Spread \partial x} * \Delta Spread * \Delta x}_{} + \ldots$$

Delta hedge spread risk in proportional to discounted EPE curve.

Determines <u>notional value</u> of credit default swaps to buy.

Delta hedge potential changes in market rates (Interest rates and implied vol) in proportion to CP spread.

Determines <u>notional value of interest rate option</u> need to buy to hedge interest rate risk and implied vol risk (or more general hedges needed to hedge more general market risk).

Potential risk of correlation of change in spread and changes in market rates.

Determines sensitivity to correlation risk, which may not be possible to delta hedge.

Correlation can be right or wrong way.

Effectively have transformed counterparty credit risk into market risk.