Algorithmic Trading: A Buy-Side Perspective

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Outline

- Overview of algorithmic trading: (30 minutes)
    - Definitions
    - The buy-side vs. sell-side views
    - Explicit and implicit costs
    - Market impact and implementation shortfall
- Recent developments (45 minutes)
    - Building impact models with public data
    - Portfolio optimization with the market impact costs
    - Multi-period portfolio optimization with transaction costs and constraints
- Final thoughts and questions (15 minutes)

What Is Algorithmic Trading?[1]

General definition: *Trading in an automated fashion according to a set of rules*

Includes the following functions:

- Optimal execution — "New York definition"
- Smart order routing
- Program trading
- Market impact modeling
- Execution risk analytics
- Market making
- Statistical trading or statistical arbitrage
- The "exploitation" of market microstructure effects
- Cost aware portfolio construction
- . . . and more

(Market making, statistical trading or statistical arbitrage, and the "exploitation" of market microstructure effects: "Chicago definition")

Algorithmic Trading: Buy-Side vs. Sell-Side

Buy-side

Allocation of capital to *maximize expected portfolio value subject to risk budget and constraints*

- Alpha models
- Market impact models
- Transaction cost aware portfolio construction
- Optimal execution from a portfolio perspective
- Monitoring of risk and leverage
- . . . and more

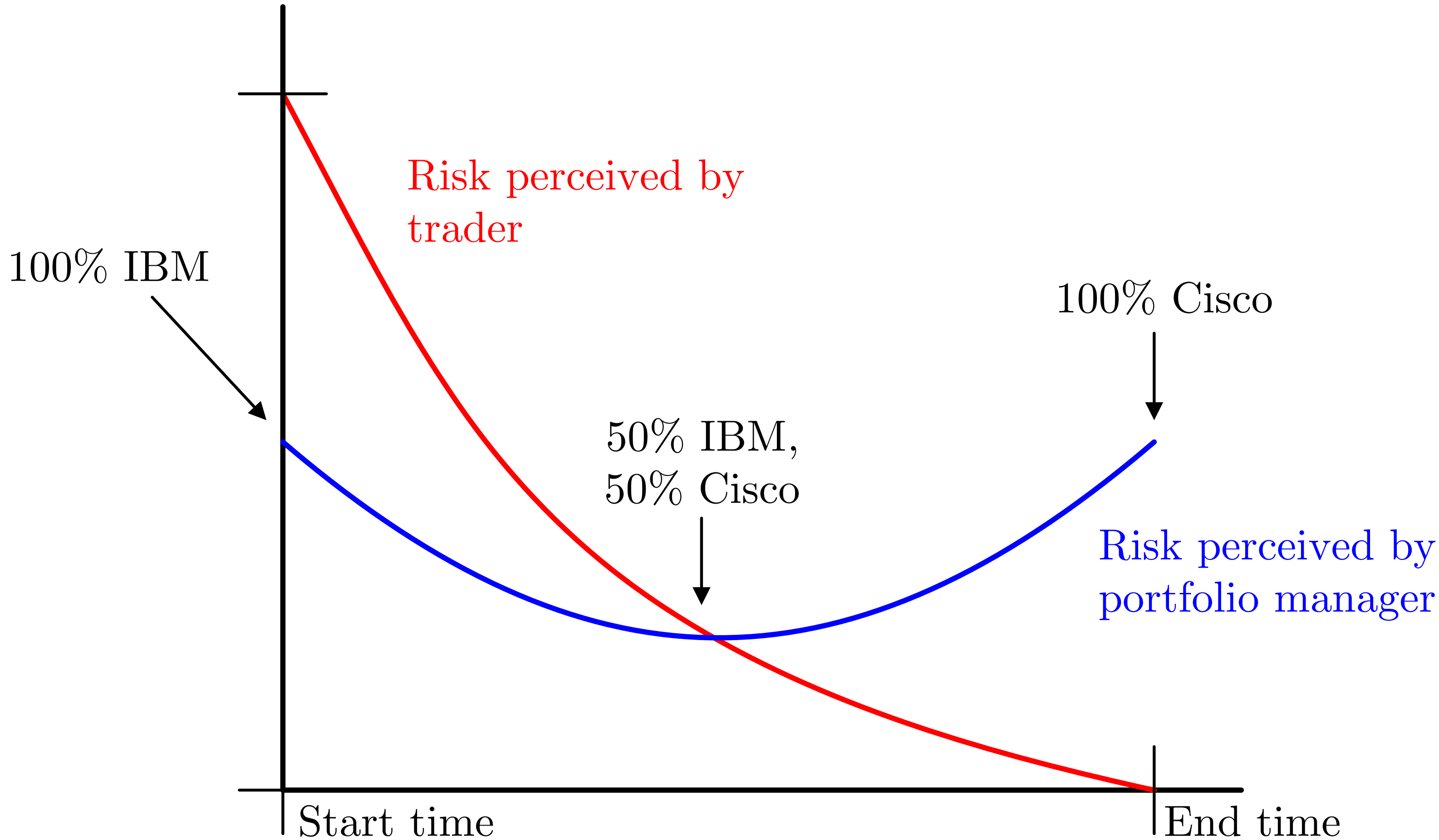
Sell-side

Executing trades to *minimize risk adjusted cost of execution*

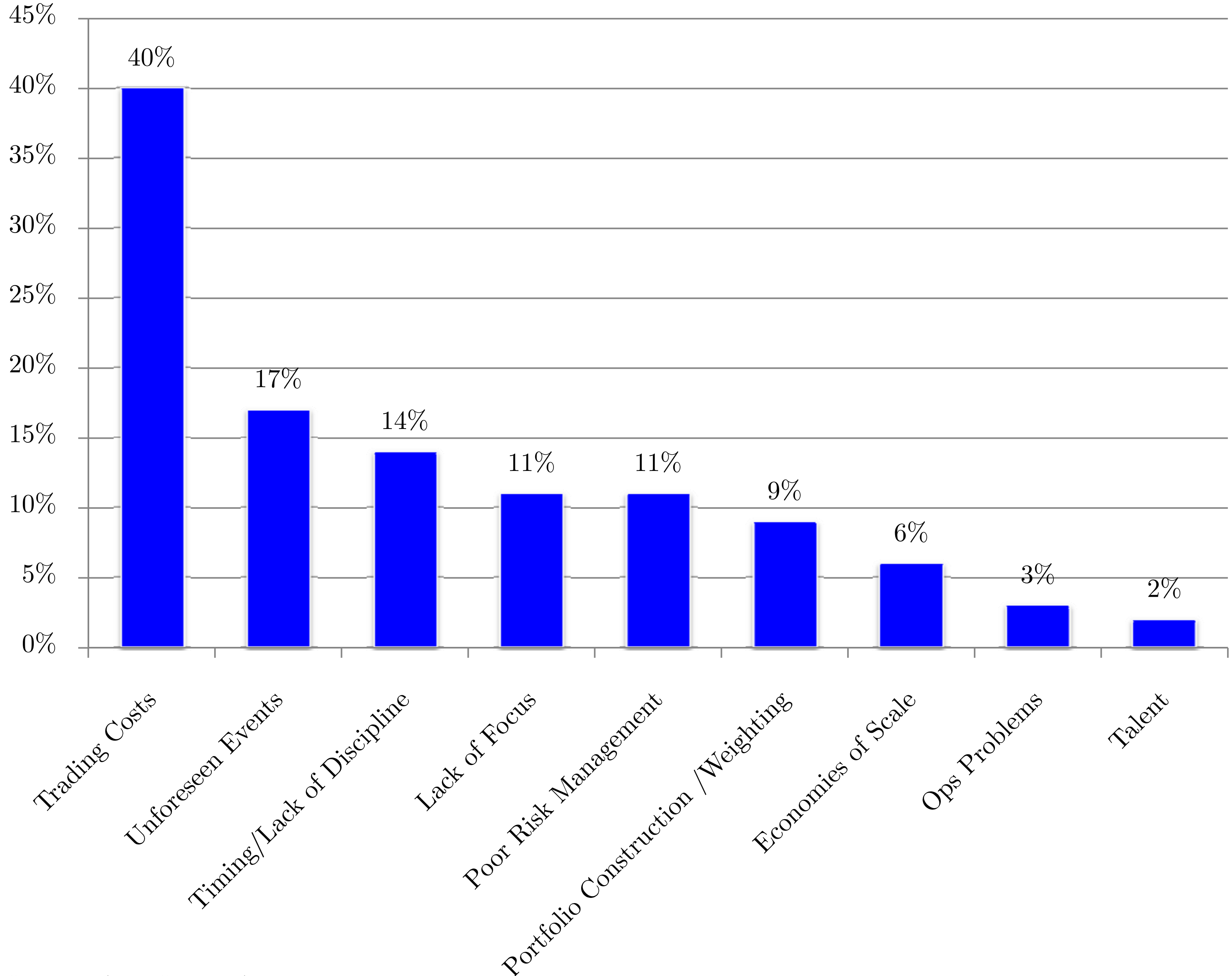
- Optimal execution
- Smart order routing
- Direct market access
- Principal bid programs
- Pre- and post-trade analytics
- . . . and more

The Views of Traders vs. Portfolio Managers

Example: Exchanging IBM for Cisco



Where is Alpha Lost? (Domestic Equity Managers)



Source: TABB Group, "Alternative Investments 2007"

Transaction Costs

- *True* transaction costs are not measurable

  (a) Explicit costs: commissions, bid-ask spreads, taxes, and foreign exchange costs; (b) Implicit costs: market impact, opportunity cost

- Implementation shortfall = trade price - decision price + commissions (Treynor (1981), Perold (1988))

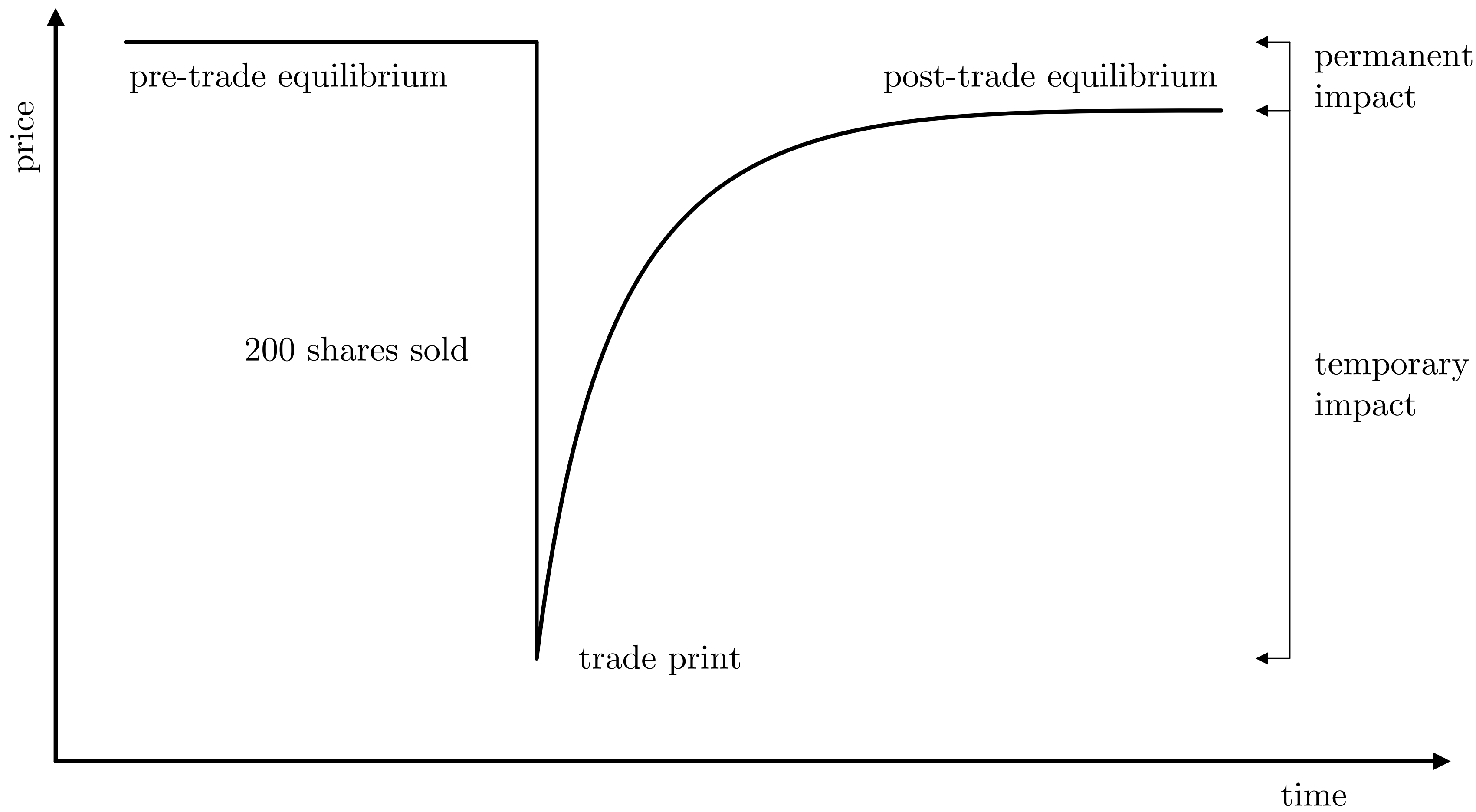
- Desirable characteristics of a T-cost model:

  - Unbiasedness: On average the forecasts of market impact should match observed costs

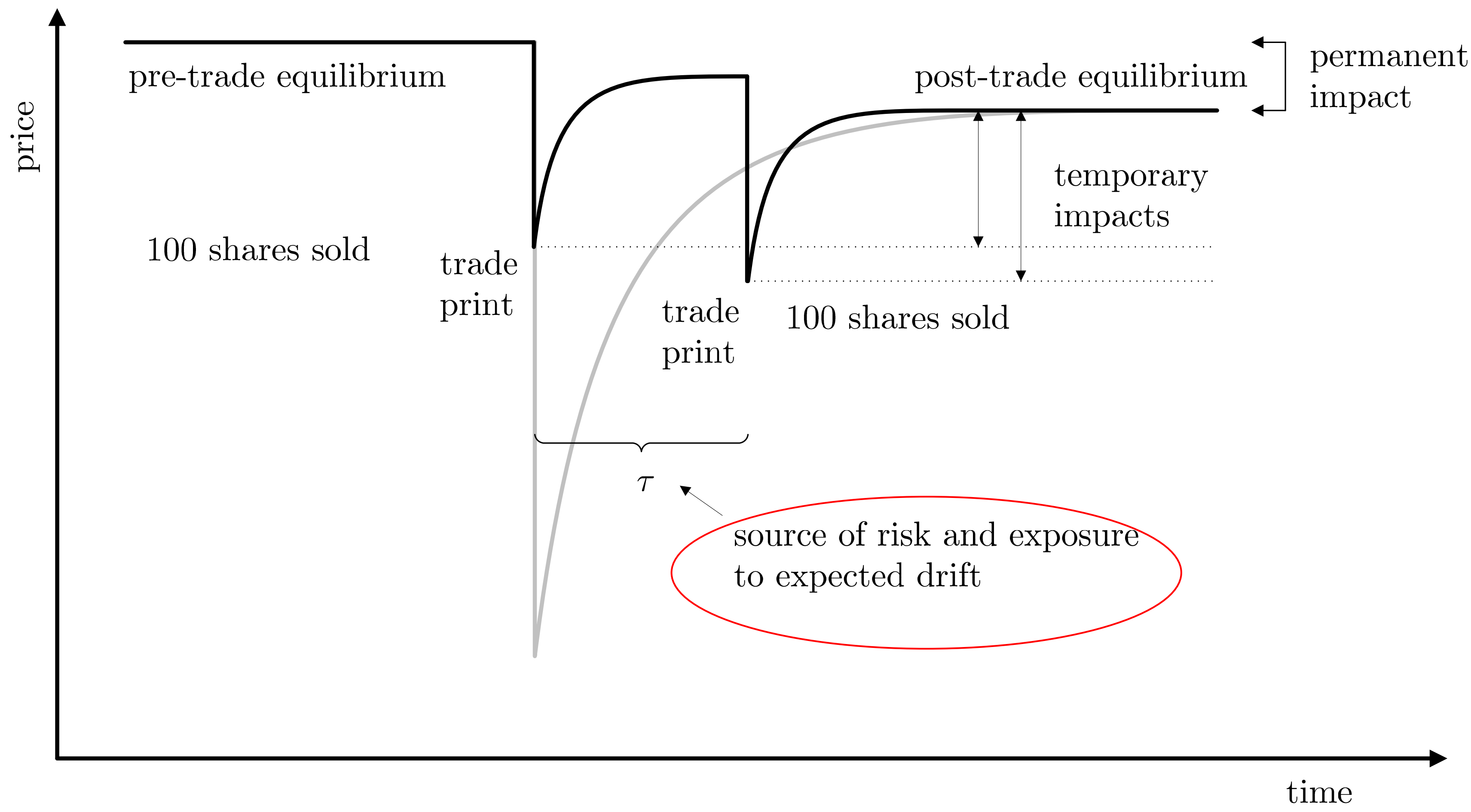
  - Low error: The absolute difference between the forecasts and the observations as a percentage of the forecast should be small

  - High explanatory power: The model should accurately predict when the shortfall/impact of a trade will be high or low

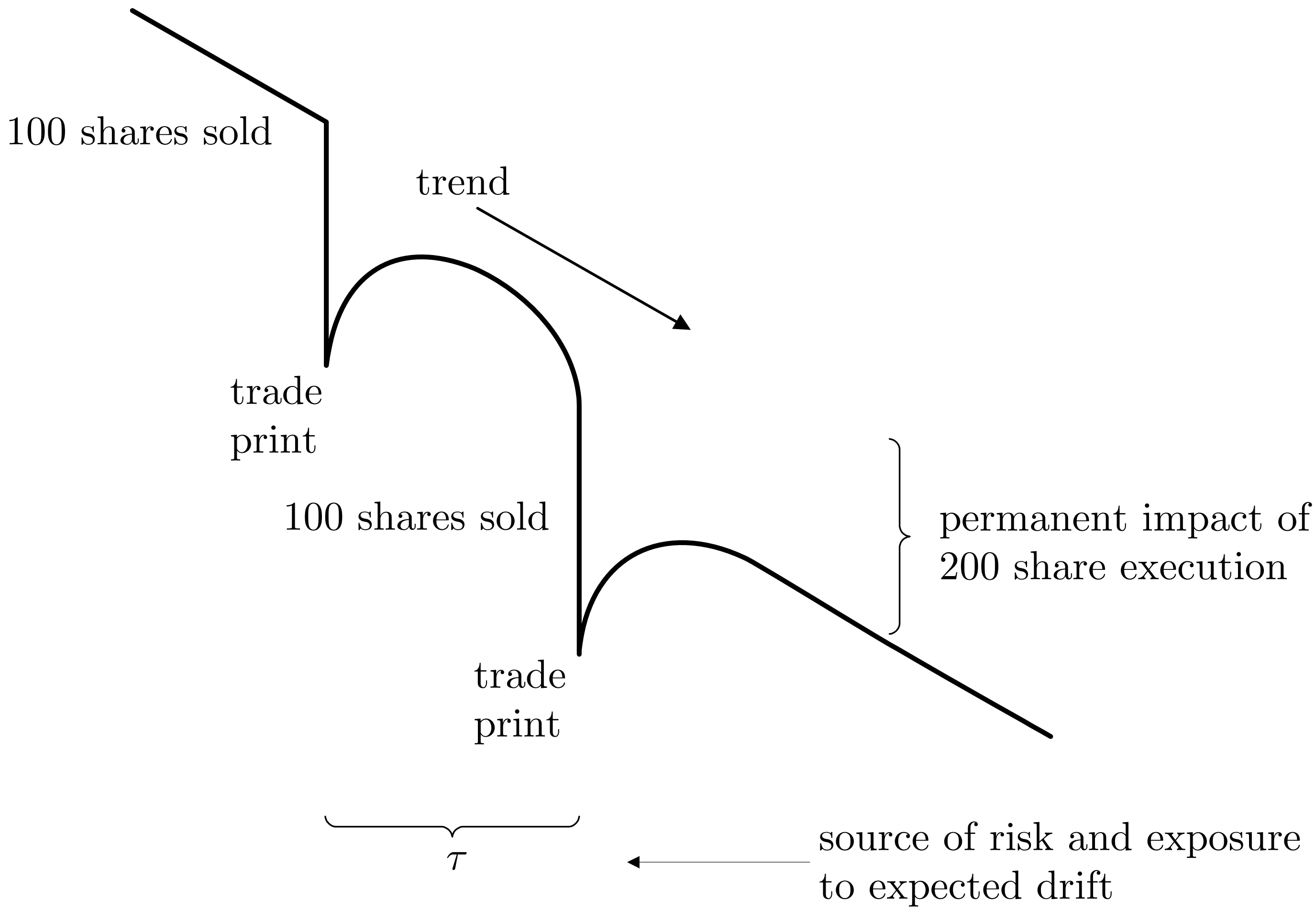
Idealized Market Impact Model – Selling of 200 Shares (1 Trade)



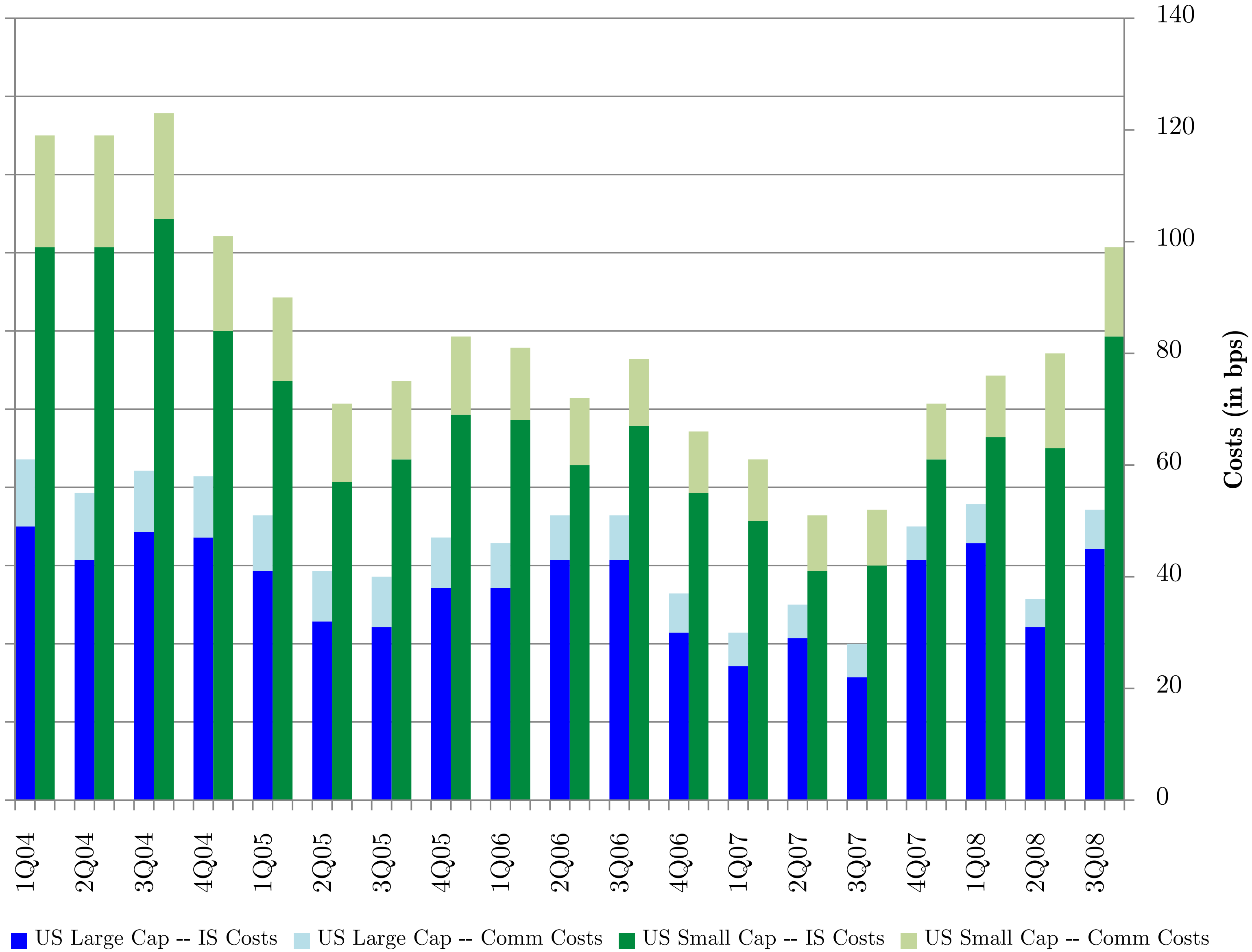
Idealized Market Impact Model – Selling of 200 Shares (2 Trades)



Idealized Market Impact Model – Selling of 200 Shares (2 Trades) with Alpha

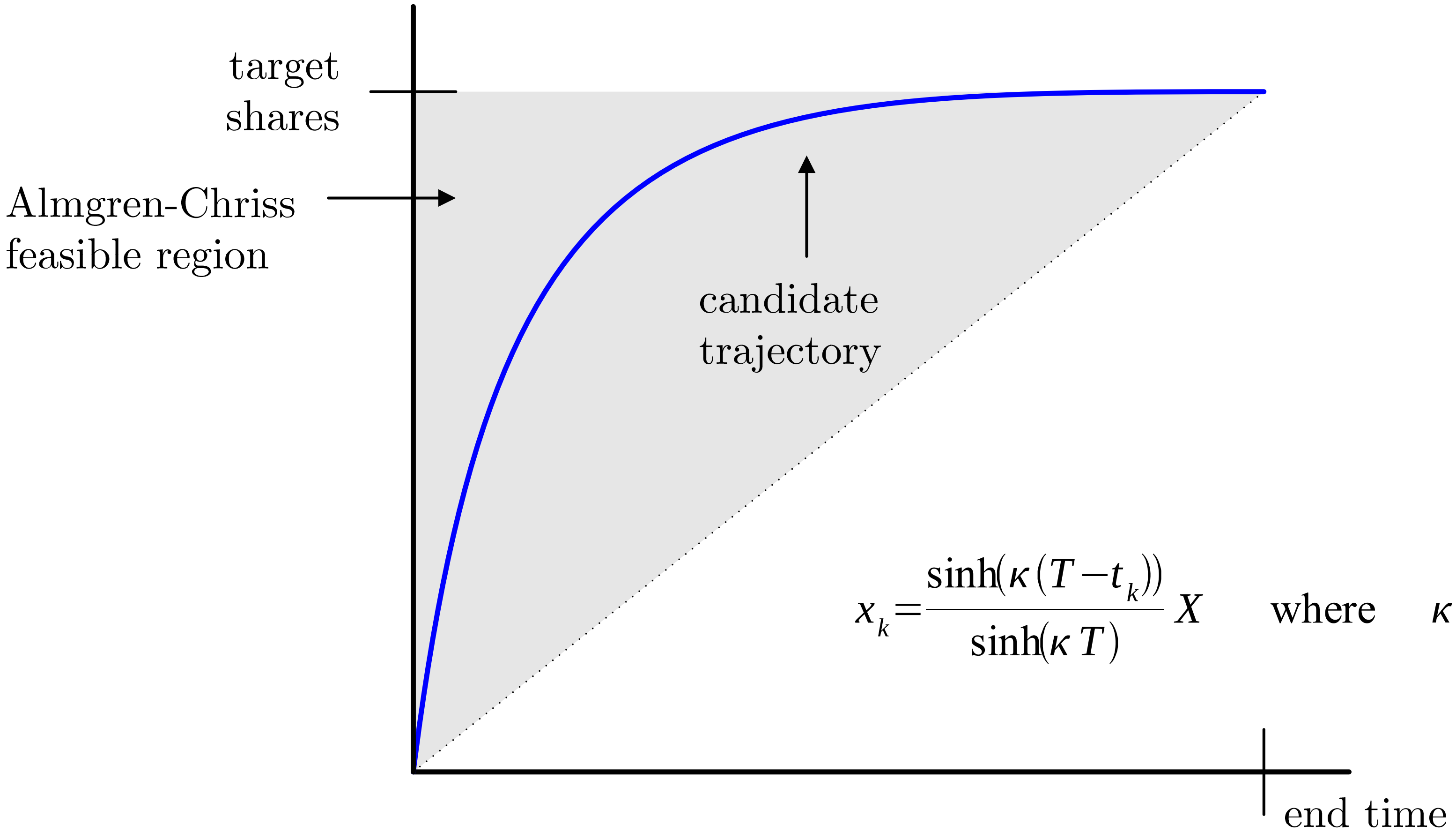


Transaction Costs (Large vs. Small Cap, 1stQ 2004 – 3rdQ, 2008)[2]



Source: ITG Global Trading Costs Review, Q3 2008

Optimal Execution à la Almgren and Chriss (2000)



$$x_k = \frac{\sinh(\kappa (T - t_k))}{\sinh(\kappa T)} X \quad \text{where} \quad \kappa = \sqrt{\frac{\lambda \sigma^2}{\eta}}$$

Building Impact Models with Public Data[3]

Trades and Quotes database (TAQ):

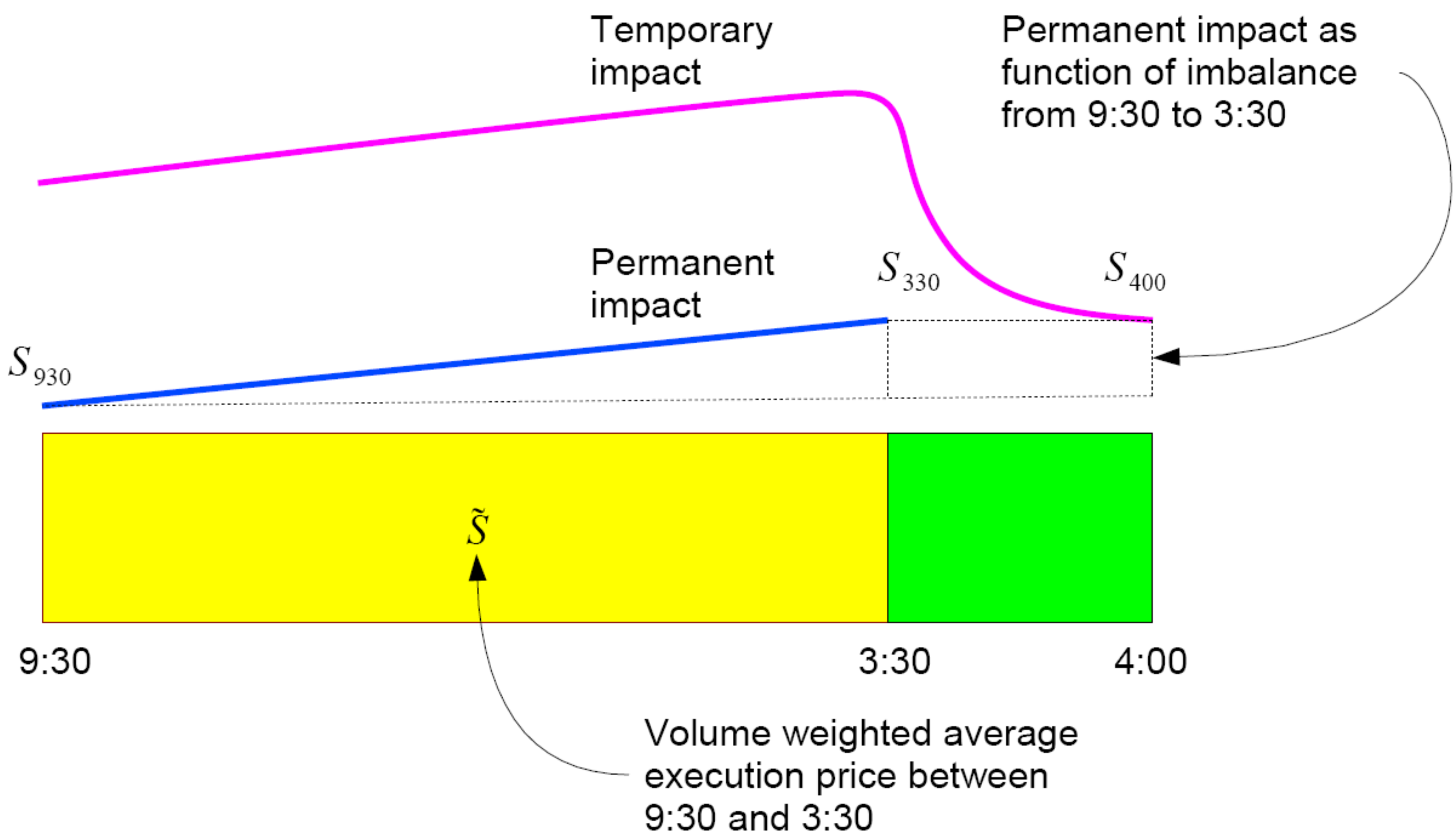
Disadvantages

- Does not include parent orders
- We have to infer trade direction
    - Introduces a bias – we overestimate impact
- We use order imbalance as a proxy for trades

Advantages

- Huge data set:
    - Includes trades and quotes of the whole market (NYSE and NASDAQ)
    - Liquid and illiquid stocks
- None of the biases typically introduced by high alpha institutional data sets
- All "trades" same length
- No heteroskedasticity error correction needed

Separate Temporary and Permanent Impacts (1/2)



$$I_t^{perm} = \frac{S_{400} - S_{930}}{2}$$

$$I_t^{temp} = \tilde{S} - S_{930} - I_t^{perm}$$

Separate Temporary and Permanent Impacts (2/2)

Separate temporary and permanent impact as in Almgren et al. (2005):

- Assume linear permanent impact[4]
- Perform regression to find permanent and temporary impact parameters (β, γ, η):

$$I_t^{perm} = \gamma \cdot \sigma_t \cdot \left| \frac{X_t}{V_t} \right| + \varepsilon_t^{perm}$$

$$I_t^{temp} = \eta \cdot \sigma_t \cdot sign(X_t) \cdot \left| \frac{X_t}{(6\,/\,6.5) V_t} \right|^{\beta} + \varepsilon_t^{temp}$$

X_t: imbalance between 9:30 a.m. and 3:30 p.m.

V_t: average daily volume

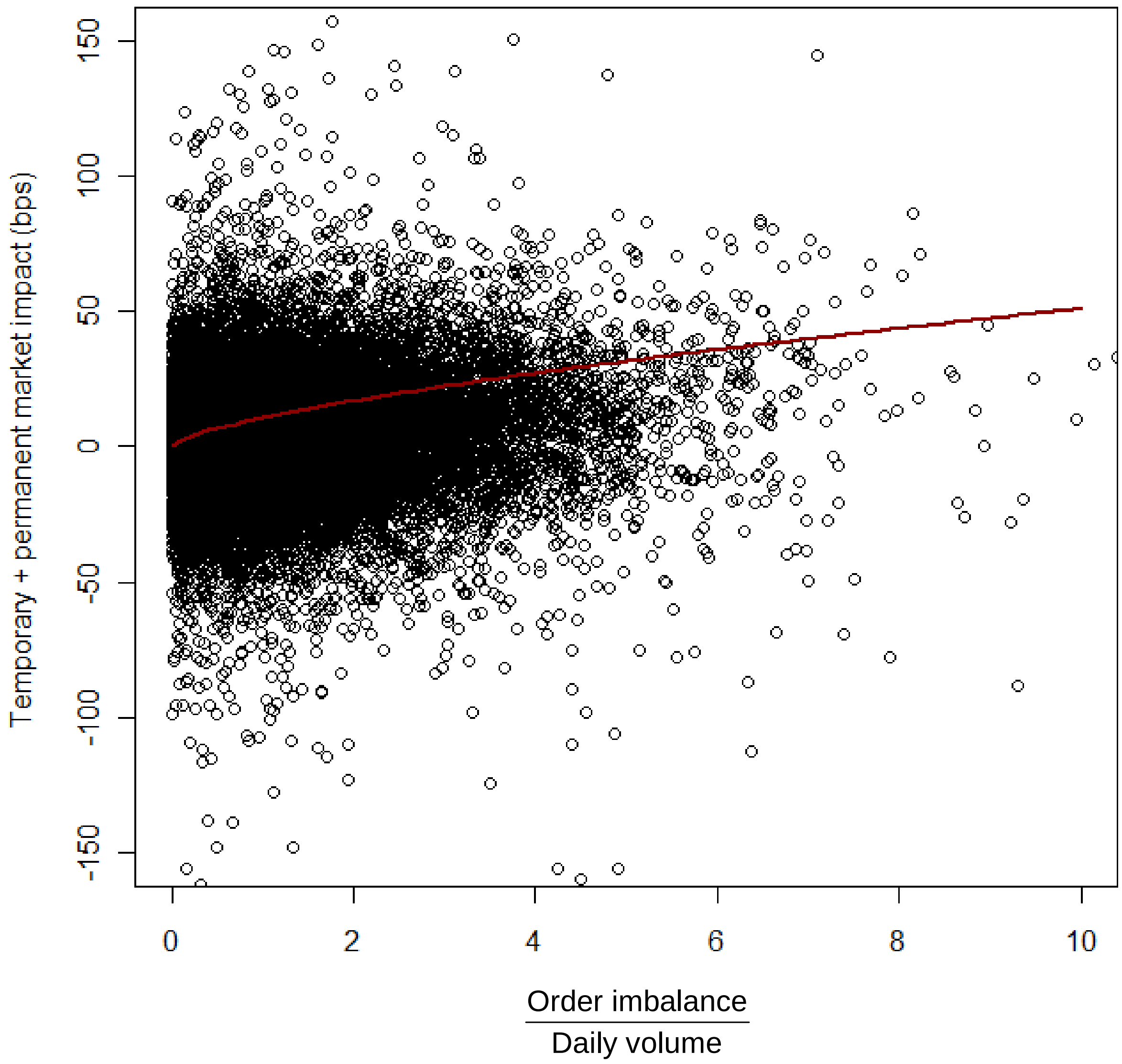
σ_t: standard deviation of 2-min returns scaled to 1 day

$$I_t^{perm} = \frac{S_{400} - S_{930}}{2}$$

$$I_t^{temp} = \tilde{S} - S_{930} - I_t^{perm}$$

Market Impact

$$\beta = 0.63 \pm 0.054 \qquad [0.60 \pm 0.038]$$
$$\gamma = 2.31 \pm 0.073 \qquad [0.31 \pm 0.041]$$
$$\eta = 0.22 \pm 0.028 \qquad [0.14 \pm 0.006]$$



Transaction costs extensions of the mean-variance framework are typically of the form

$$\begin{aligned}&\max_{w} w^T\mu - \lambda w^T\Sigma w - TC(x)\\&\text{s.t.}\quad w^T e + TC(x) \le w_{prev}^T e\\&w \in C\end{aligned}$$

where

w: portfolio weights;

Σ: covariance matrix of returns;

$x = w - w_{prev}$: trade;

$e = (1,1,...,1)^T$

μ: vector of expected returns

λ: risk aversion coefficient

C: other constraints

Transaction costs for trade x:

$$TC_i(x_i) = \max\left\{a_i \cdot |x_i|,\quad b_i \cdot |x_i|^2 + c_i \cdot |x_i|^{1+\beta}\right\}$$

Practical Considerations

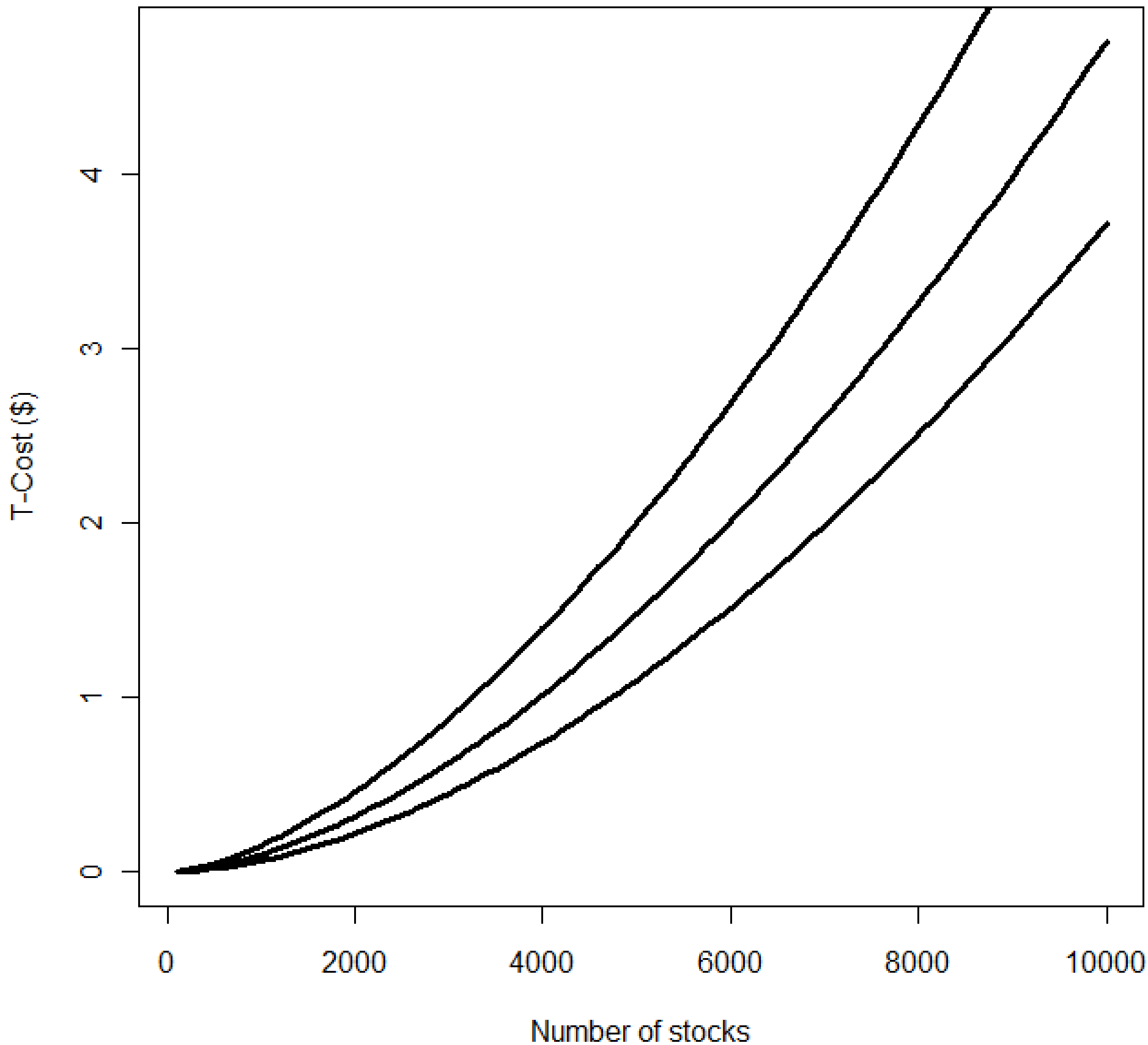
- Transaction costs models often involve nonlinear functions
- Software for general nonlinear optimization problems available, *but* computational time required for solving such problems is often too long for realistic investment management applications (large universes with thousands of assets)
- Efficient and reliable software is available for linear (LP), quadratic (QP), and second-order cone programs (SOCP)

→ Can approximate more complex nonlinear optimization problem by simpler problems that can be solved quickly (piecewise linear approximations)

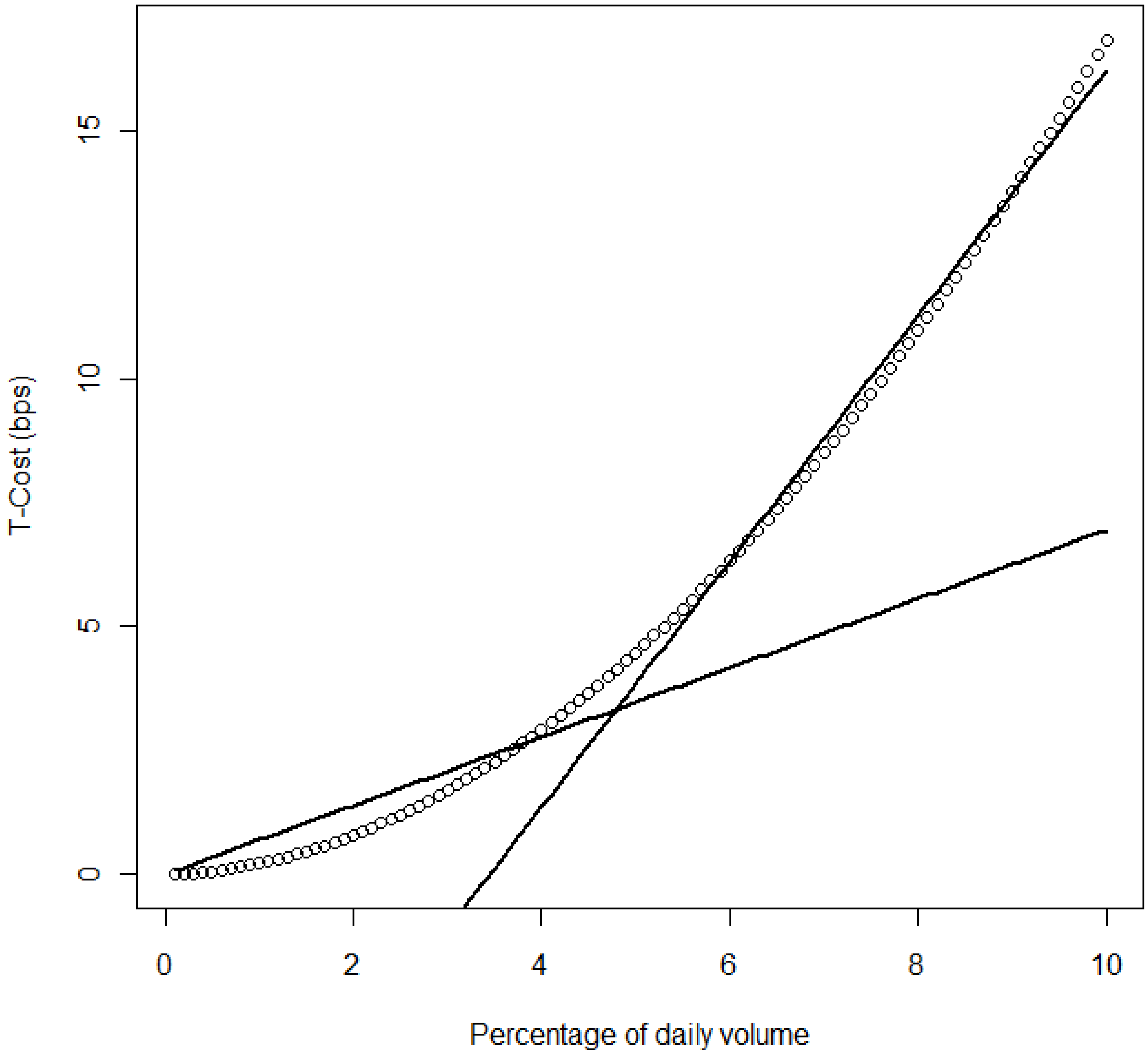
Related Work

- Early work:
    - Pogue (1970), Schreiner (1980), Perold (1984)
- Linear costs:
    - Adcock & Meade (1994), Mitchell & Braun (2002)
- Piecewise linear costs:
    - Potaptchik, Tuncel & Wolkowicz (2008)
- Fixed and linear costs:
    - Lobo, Fazel, & Boyd (2000)
- Nonlinear costs (convex):
    - Best & Hlouskova (2007a/b, 2008)
- Nonlinear costs (market impact model):
    - Axioma & Goldman Sachs (2008), Borkovec, Domowitz et al. (2009)

Estimated Market Impact Cost Function (with Standard Errors)

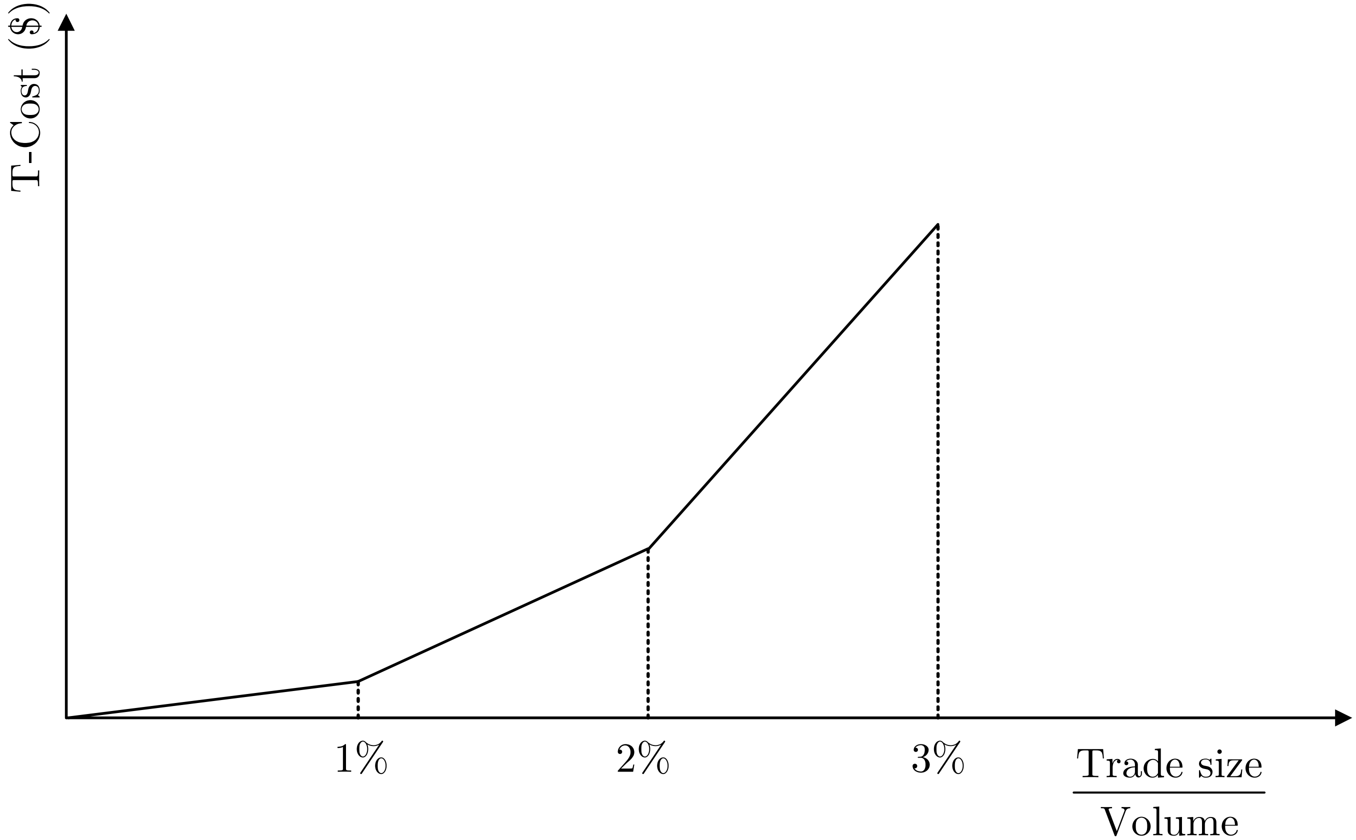


Piecewise Linear Approximation (2 Pieces)



(a) Type of fit; (b) No. of segments (c) Placement of breakpoints

Piecewise Linear Approximation: Simple Model



$$TC(x) = \begin{cases} s_1 x, & 0 \le x \le 1\% \cdot \text{Vol} \\ s_1 \left(1\% \cdot \text{Vol}\right) + s_2 (x - 1\% \cdot \text{Vol}), & 1\% \cdot \text{Vol} \le x \le 2\% \cdot \text{Vol} \\ s_1 \left(1\% \cdot \text{Vol}\right) + s_2 (1\% \cdot \text{Vol}) + s_3 (x - 2\% \cdot \text{Vol}), & 2\% \cdot \text{Vol} \le x \le 3\% \cdot \text{Vol} \end{cases}$$

where $s_1 < s_2 < s_3$ are the (known) slopes of the three linear segments

A "QP Friendly" Formulation

$$TC(x) = \sum_{i=1}^{N} \left(s_{1,i} \cdot y_{1,i} + s_{2,i} \cdot y_{2,i} + s_{3,i} \cdot y_{3,i} \right), \qquad s_1 < s_2 < s_3$$

where[6]

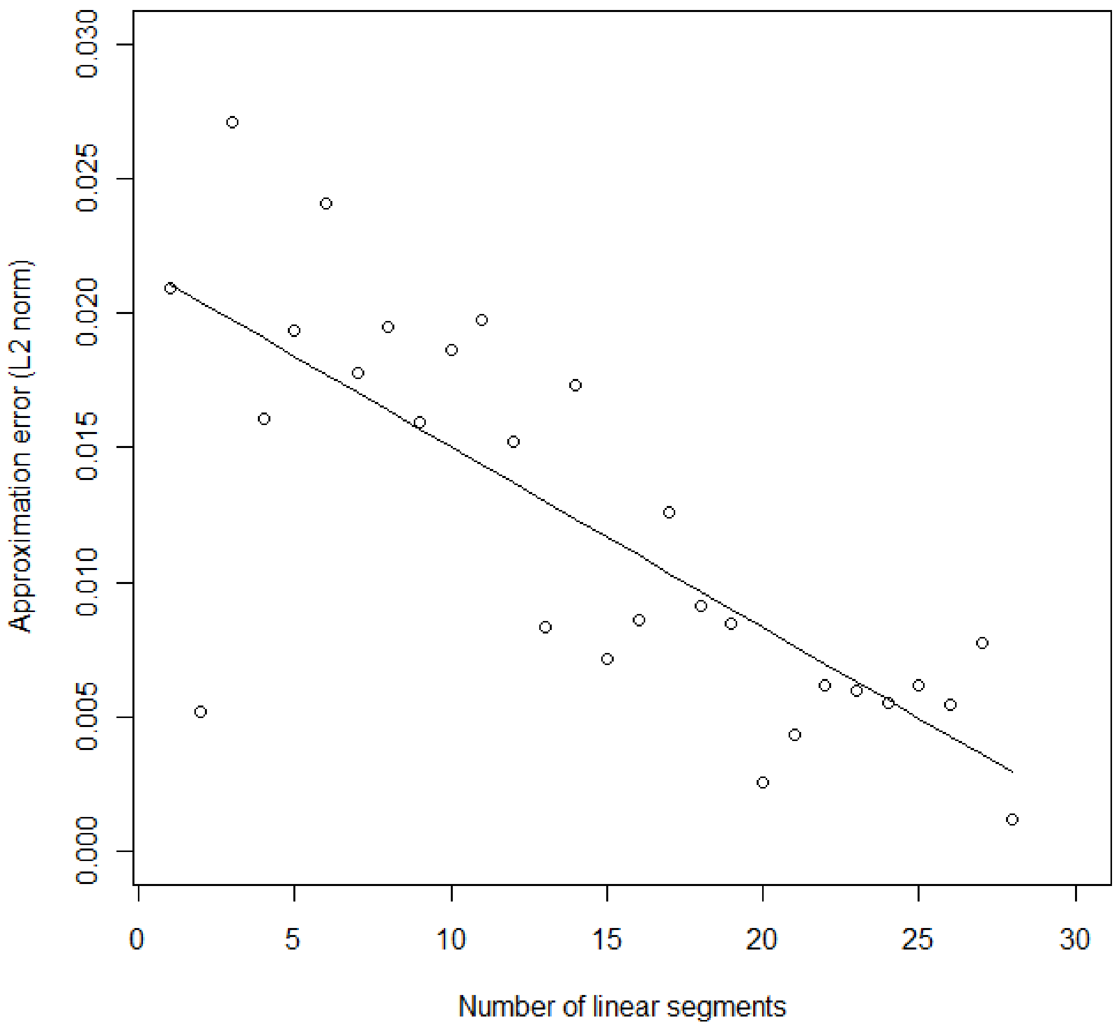
$$\begin{aligned} 0 \le y_{1,i} \le 0.15 \cdot \mathrm{Vol}_i \\ 0 \le y_{2,i} \le 0.25 \cdot \mathrm{Vol}_i \\ 0 \le y_{3,i} \le 0.15 \cdot \mathrm{Vol}_i \end{aligned}$$

and

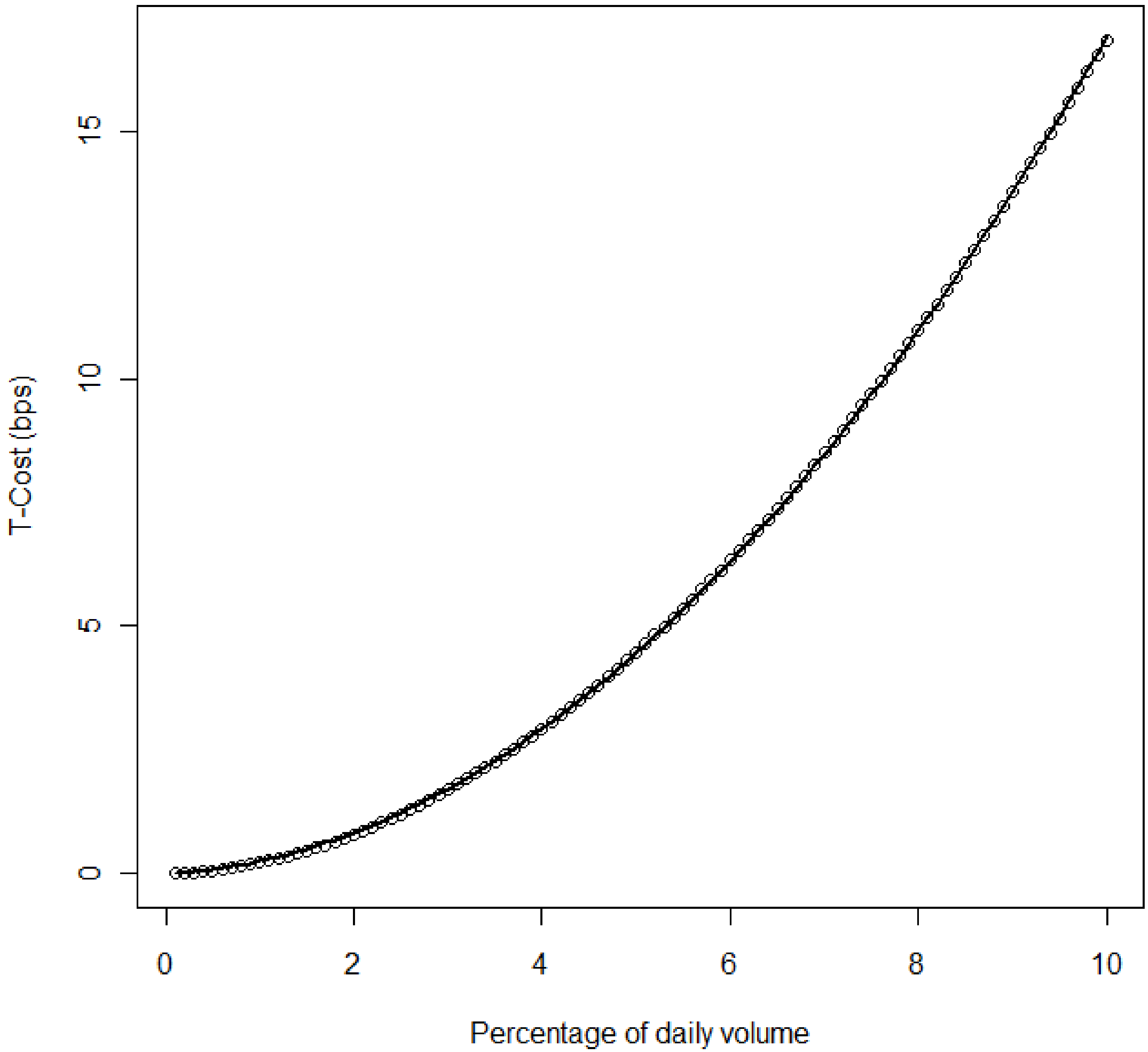
$$\left| w_i - w_{prev,i} \right| = y_{1,i} + y_{2,i} + y_{3,i}$$

- Converges to the nonlinear impact function by increasing the number of line segments
- The optimization may become "unstable" if too many segments are used[7, 8]

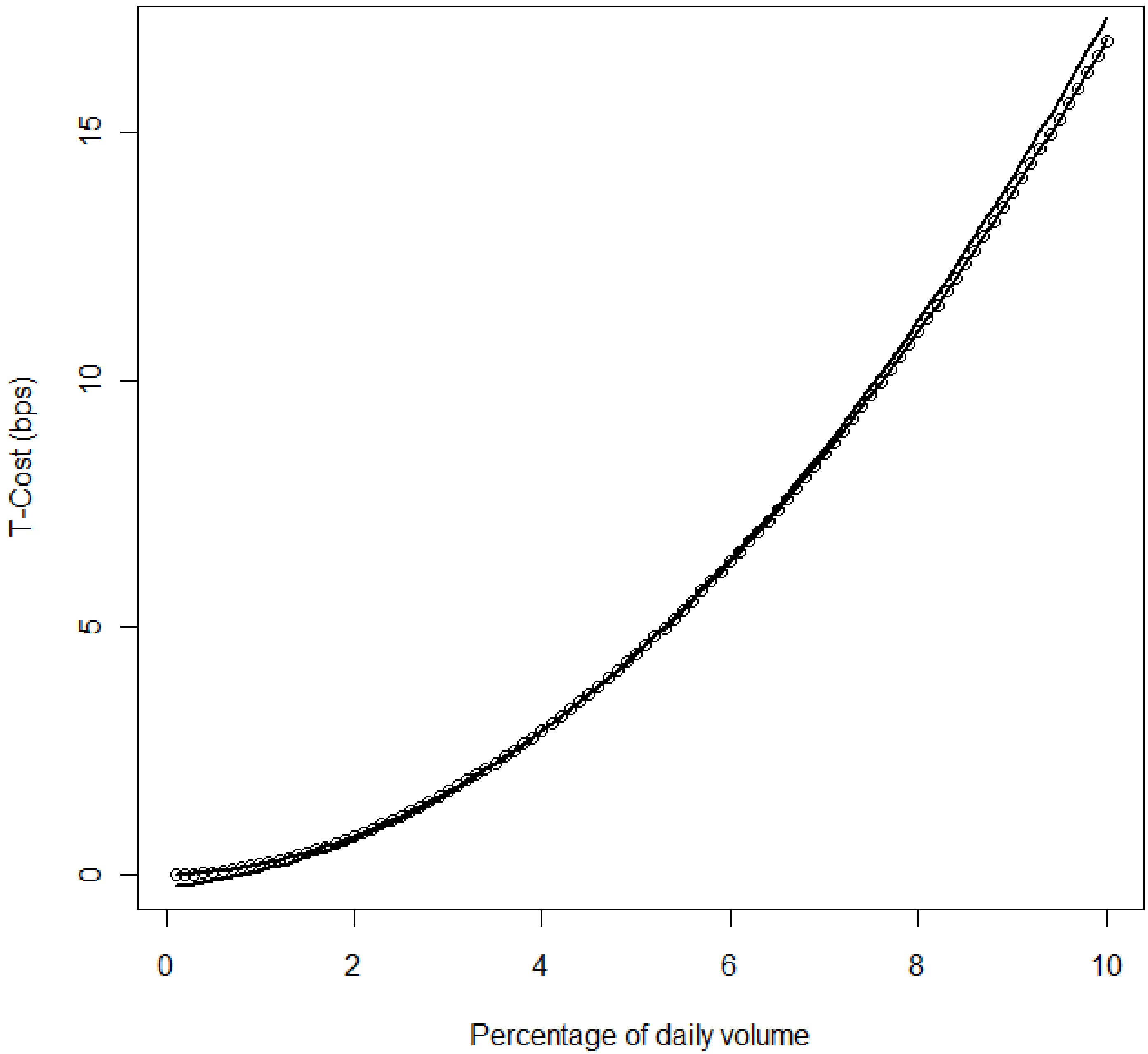
Piecewise Linear Approximation: Convergence



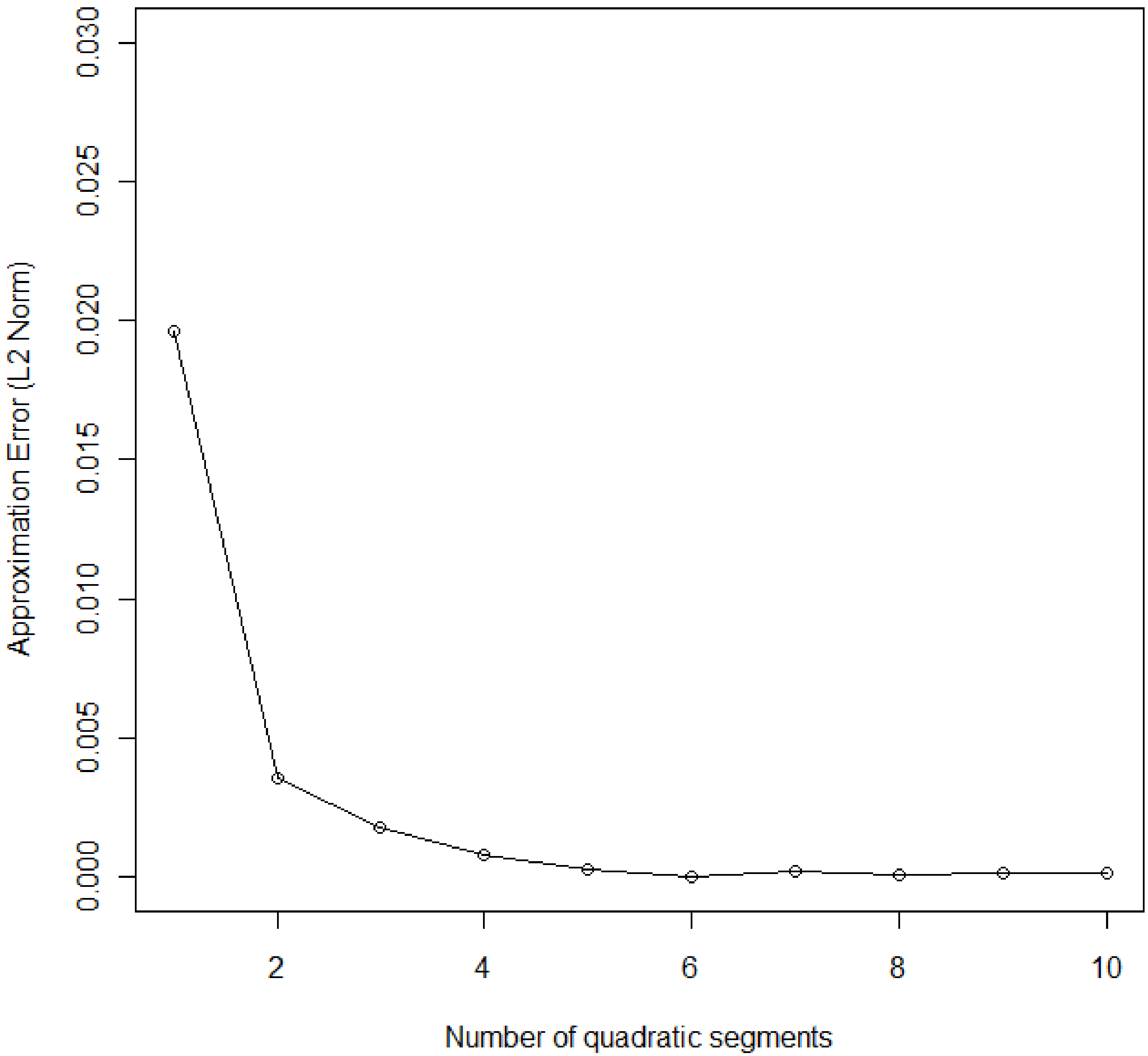
Quadratic Approximation (1 Piece)



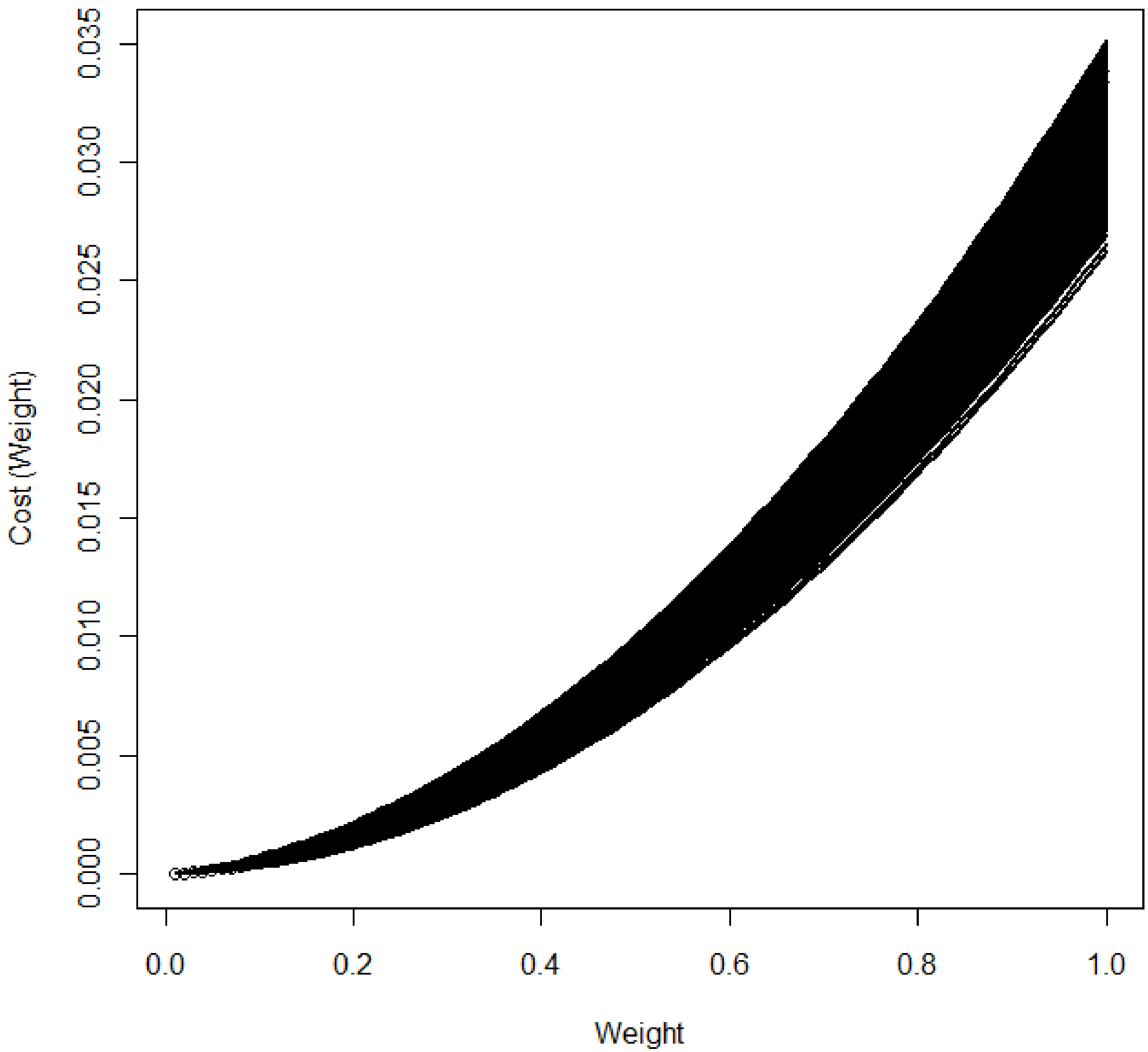
Quadratic Approximation (2 Pieces)



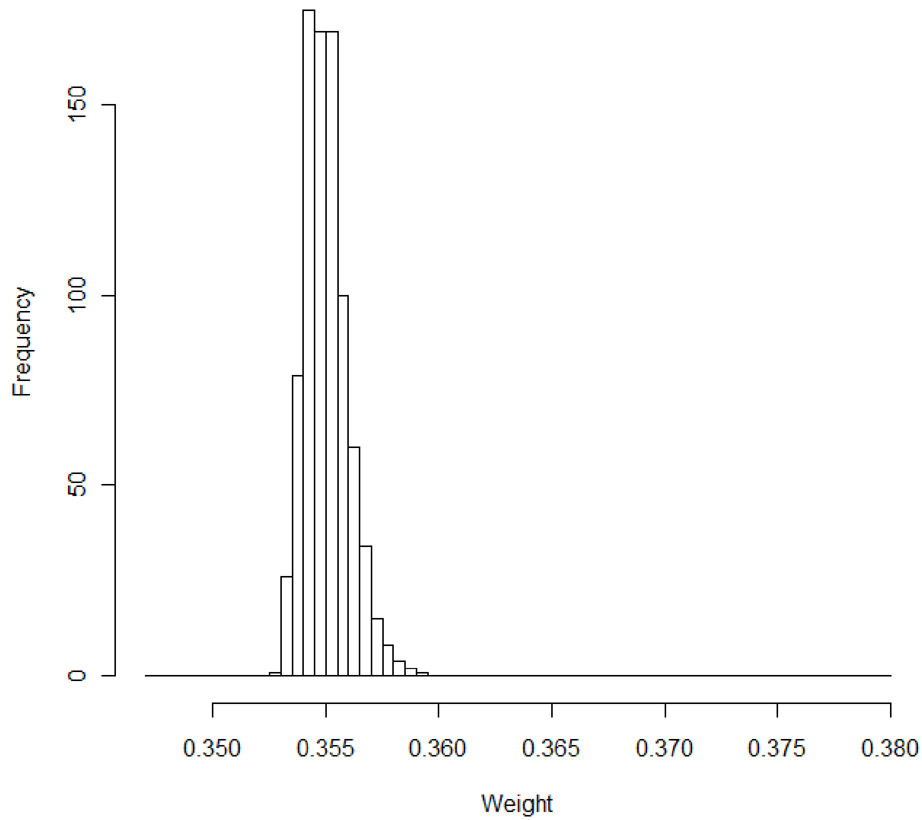
Quadratic Approximation: Convergence



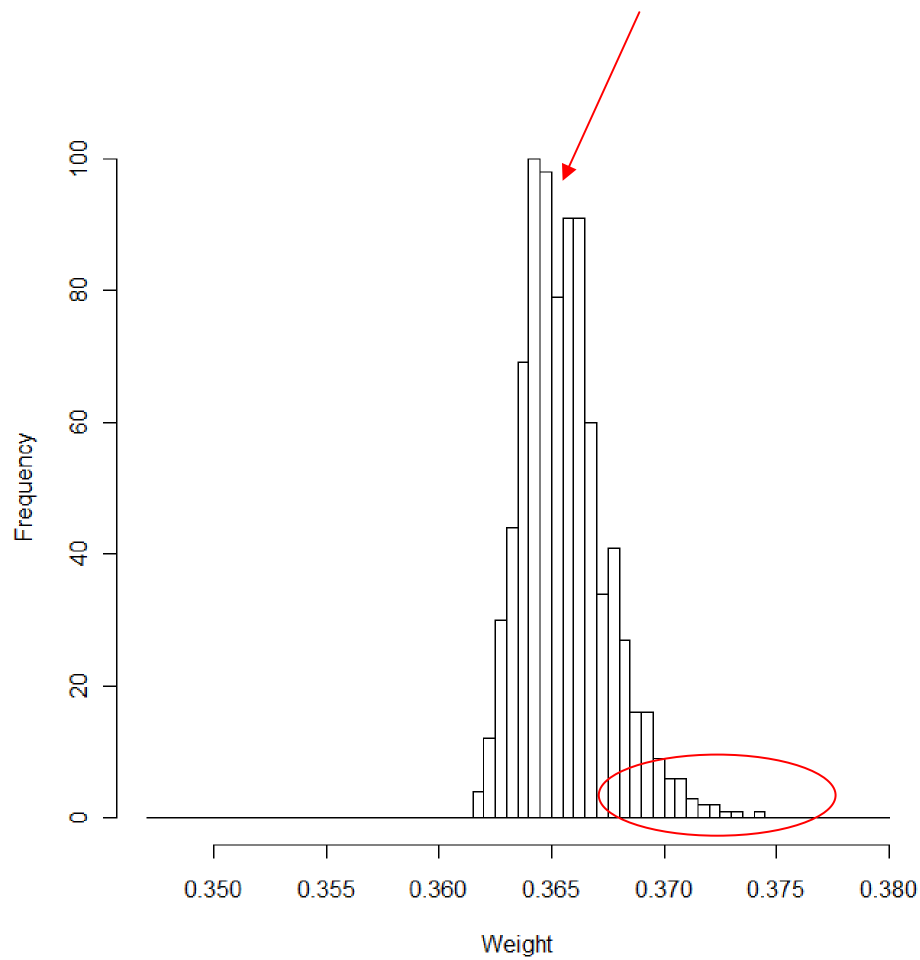
A Simulation Experiment (1/3)



A Simulation Experiment (2/3)



Nonlinear: 1st weight



Linear (1 Piece): 1st weight

A Simulation Experiment (3/3): Results

	Non-linear	Linear approximation (number of pieces)				Quadratic approximation (number of pieces)		
		1	**2**	**4**	**13**	**1**	**2**	**3**
Mean	0.355	0.366	0.351	0.357	0.353	0.364	0.362	0.362
Median	0.355	0.365	0.351	0.355	0.353	0.364	0.362	0.355
Std Dev	0.001	0.002	0.001	0.002	0.001	0.002	0.001	0.001
Skew	0.786	0.841	0.781	0.495	0.795	0.294	0.732	0.758
Kurtosis	1.009	1.175	0.933	0.298	0.986	-0.287	1.026	1.074
Max	0.359	0.374	0.358	0.366	0.358	0.371	0.368	0.361
Min	0.353	0.362	0.348	0.351	0.352	0.359	0.359	0.353

- In practice, 2-3 linear or 1-2 quadratic segments are sufficient to resolve the impact function to the order of the *estimation error* of the market impact model

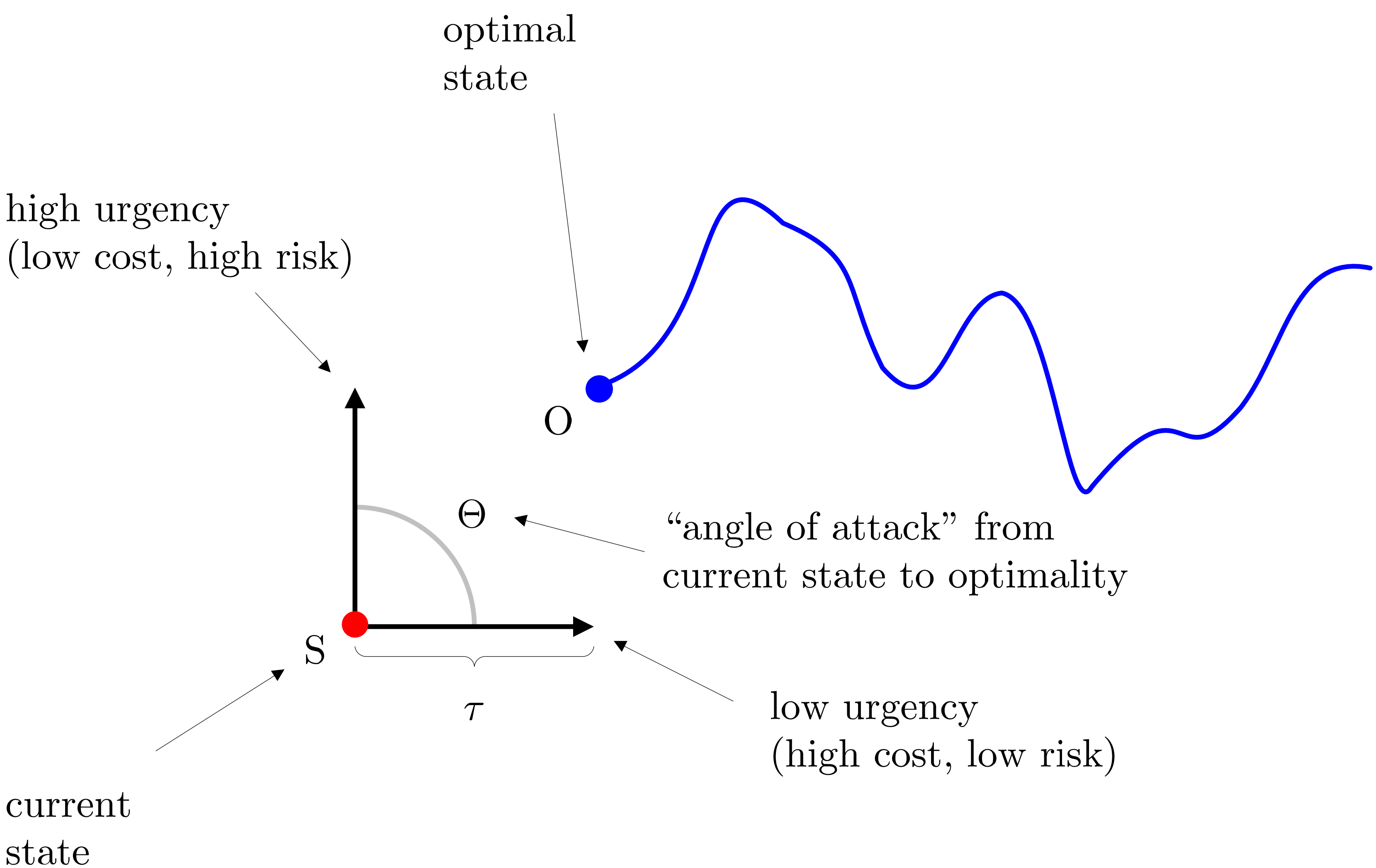
Multi-Period Portfolio Optimization with Transaction Costs and Alpha Decay[9]

- In practice multi-period models are seldom used
  - Difficulty of estimating return/risk for multiple periods
  - Computationally burdensome, especially if the universe is large
- Instead, it is common to use a single-period (myopic) model and rebalance from period to period

- Why use multi-period models?
  - Alpha decay
  - Our trading today impacts asset prices in the future
  - Minimize our trading costs
  - Portfolio in/out-flow (additional investments, redemptions, taxes, etc.)

Related Work

- Merton (1969, 1990)
  - The investment-consumption problem
- Kritzman, Myrgren and Page (2007)
  - Portfolio transitions
- Engle and Ferstenberg (2007)
  - "Execution risk", the interplay between transaction costs and portfolio risk
- Grinold (2006), Gârleanu and Pedersen (2009)
  - Interplay between alpha-decay and transaction costs (only temporary one-period impact)



optimal
state
high urgency
(low cost, high risk)
O
Θ
"angle of attack" from
current state to optimality
S
τ
low urgency
(high cost, low risk)
current
state

Multi-Period Portfolio Optimization and Stochastic Optimal Control

An optimal control problem consists of the following components:

- Process dynamics
  - Asset returns, alpha decay
  - Permanent and temporary market impact
- Observable quantities
- Cost function

Note:

- In general, optimal control problems lead to complicated PDEs (Hamilton-Jacobi-Bellman equations)
- Large dimension (number of assets)

Process Dynamics (1/2)

- Returns

$$r_{t+1} = \alpha_{t+1} + \varepsilon^r_{t+1},$$

where $r_{t+1} \equiv p_{t+1} - p_t$, $E_t(\varepsilon^r_{t+1}) = 0$, and $Var_t(\varepsilon^r_{t+1}) = \Sigma$

- Alpha driven by mean-reverting factors

$$\begin{aligned} \alpha_t &= Bf_t + \varepsilon^\alpha_t \\ \Delta f_{t+1} &= -Df_t + \varepsilon^f_{t+1} \end{aligned},$$

where $f_t \in \mathbb{R}^K, B \in \mathbb{R}^{S\times K}, D \in \mathbb{R}^{K\times K}$, $E_t(\varepsilon^\alpha_{t+1}) = E_t(\varepsilon^f_{t+1}) = 0$, $Var_t(\varepsilon^f_{t+1}) = \Sigma^f$, and $Var_t(\varepsilon^\alpha_{t+1}) = \Sigma^\alpha$

We observe that if $\varepsilon^\alpha_t \equiv 0$

$$\begin{aligned} \alpha_t &= Bf_t \\ &= B(I-D)f_{t-1} + \varepsilon^f_t \\ &= \alpha_{t-1} - BDf_{t-1} + \varepsilon^f_t \end{aligned}$$

Process Dynamics (2/2)

- Temporary and permanent impact

$$\begin{aligned}\alpha_t &= Bf_t + \varepsilon_t^\alpha + \underbrace{\Pi\Delta x_t}_{\text{permanent}} + \underbrace{H\Delta x_t}_{\text{temporary}} - H\Delta x_{t-1} \\ &= Bf_t + \varepsilon_t^\alpha + (\Pi + H)\Delta x_t - H\Delta x_{t-1}\end{aligned}$$

Process Dynamics as a State Space Model

Consider the "augmented state"

$$s_t = \begin{pmatrix} f_t \\ x_t \\ h_t \end{pmatrix}$$

and observe that

$$s_t = \begin{pmatrix} I - D & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & G \end{pmatrix} s_{t-1} + \begin{pmatrix} 0 \\ I \\ (I-G)H \end{pmatrix} \Delta x_t + \begin{pmatrix} \varepsilon_t^f \\ 0 \\ \varepsilon_t^h \end{pmatrix}$$

$$= \hat{A} s_{t-1} + \hat{B} \Delta x_t + \varepsilon_t$$

where $Var_{t-1}(\varepsilon_t) = \begin{pmatrix} \Sigma_f & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \Sigma_h \end{pmatrix}$

Objective Function

Multi-period optimization problem:

$$\max_{\Delta x_1, \Delta x_2, \ldots} E\left[\sum_t (1-\rho)^t (x_t^T \alpha_t - \frac{\gamma}{2} x_t^T \Sigma x_t - \frac{1}{2} \Delta x_t^T T \Delta x_t)\right]$$

where x_t are the portfolio holdings, $\rho \in (0,1)$ is a discount factor and γ is the risk aversion coefficient

Remark:

- In this model, the trade size Δx_t is our control variable
- Model contains Grinold (2006), Engle and Ferstenberg (2007), Gârleanu and Pedersen (2009) as special cases

Objective Function (2/2)

We observe that (at time t)

$$x_t^T \alpha_t - \frac{\gamma}{2} x_t^T \Sigma x_t - \frac{1}{2} \Delta x_t^T T \Delta x_t = x_t^T (B f_t + (\Pi + H)\Delta x_t - h_{t-1}) - \frac{\gamma}{2} x_t^T \Sigma x_t - \frac{1}{2} \Delta x_t^T T \Delta x_t$$

$$= \begin{pmatrix} s_t \\ \Delta x_t \end{pmatrix}^T \begin{pmatrix} R & S^T \\ S & Q \end{pmatrix} \begin{pmatrix} s_t \\ \Delta x_t \end{pmatrix}$$

$$\equiv c(s_t, \Delta x_t)$$

where

$$R = \begin{pmatrix} 0 & \frac{1}{2} B^T & 0 \\ \frac{1}{2} B & -\lambda \Sigma & -\frac{1}{2} I \\ 0 & -\frac{1}{2} I & 0 \end{pmatrix}, \quad S = \begin{pmatrix} 0 \\ \frac{1}{2}(\Pi + H) \\ 0 \end{pmatrix}, \text{ and } Q = -T$$

Note that at time T, $R_T = R$.

Multi-Period Portfolio Optimization with Market Impact as a Stochastic LQ Regulator Problem

Process dynamics:

$$s_t = \hat{A} s_{t-1} + \hat{B} \Delta x_t + \varepsilon_t$$

Objective function:

$$\max_{\Delta x_1, \Delta x_2, \ldots} E\left[\sum_{t=1}^{T-1} (1-\rho)^t c(s_t, \Delta x_t) + C_T(s_T) \right]$$

where

$$c(s, \Delta x) = \begin{pmatrix} s \\ \Delta x \end{pmatrix}^T \begin{pmatrix} R & S^T \\ S & Q \end{pmatrix} \begin{pmatrix} s \\ \Delta x \end{pmatrix}$$

$$C_T(s) = s^T R_T s$$

and $R = R^T \geq 0, S \geq 0, Q = Q^T > 0, R_T = R_T^T \geq 0$

Remarks:

- Solution through dynamic programming
- The optimal trade is linear in the state variable, i.e.

$$\Delta x_{\tau} = L_{\tau} s_{\tau}$$

- Portfolio problems with a large number of assets can be solved

The Constrained Multi-Period Portfolio Optimization Problem

Multi-period portfolio optimization with transaction costs:

$$s_t = \hat{A} s_{t-1} + \hat{B} \Delta x_t + \varepsilon_t$$

$$\max_{\Delta x_1, \Delta x_2, \ldots} E\left[\sum_{t=1}^{T-1} (1-\rho)^t c(s_t, \Delta x_t) + C_T(s_T) \right]$$

We introduce linear inequality constraints of the form

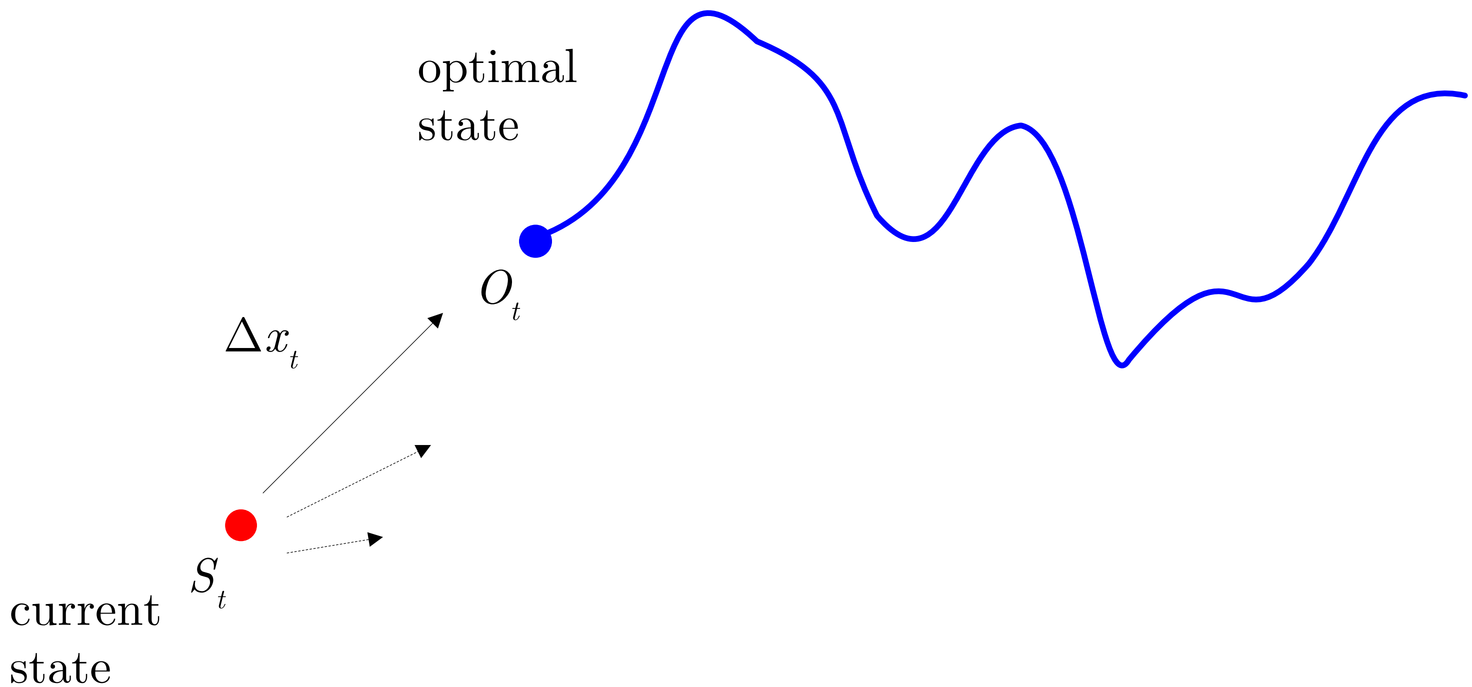
$$G_S s_t + G_{\Delta x} \Delta x_t \le g, \; t = 1, 2, 3, \ldots$$

where $G_s \in \mathbb{R}^{L \times (K+2N)}$, $G_{\Delta x} \in \mathbb{R}^{L \times N}$, and $g \in \mathbb{R}^L$

Remarks:

- Does not have a closed form solution; no Riccati equation (cf. QP)
- The deterministic counterpart (i.e. $\varepsilon_t \equiv 0$) can be shown to have a piecewise linear optimal control defined on a polyhedral partitioning of the state space
    - Can be difficult to calculate the partitioning

Solution through Receding Horizon (1/2)



Solution through Receding Horizon (2/2)

Then an approximation at time $\tau = t$ of the optimal control is found by solving the quadratic programming (QP) problem:

$$\max_{\Delta x_t, \Delta x_{t+2}, \ldots} \sum_{\tau=t}^{t+T-1} (1-\rho)^{\tau-t+1} c(s_\tau, \Delta x_\tau) + C_{t+T-1}(s_{t+T-1})$$

subject to the constraints

$$\begin{aligned} s_\tau &= \hat{A} s_{\tau-1} + \hat{B} \Delta x_\tau \\ 0 &= \hat{A} s_{t+T-1} + \hat{B} \Delta x_{t+T-1} \\ G_S s_\tau + G_{\Delta x} \Delta x_\tau &\leq g, \ \tau = t+1, t+2, t+3, \ldots \end{aligned}$$

with unknowns $s_{t+1}, \ldots, s_{t+T-1}$ and $\Delta x_{t+1}, \ldots, \Delta x_{t+T-1}$.

Final Thoughts

- Different views: traders ("sell-side") vs. portfolio managers ("buy-side")
  - Correctly assess your execution and portfolio risks
- Incorporate market impact costs into your portfolio optimizations
- Any positive expected value strategy is tradable at *some* rate of trading (fixed costs aside)
- Rebalancing less frequently does not reduce market impact costs – it often increases them
- In the next few years, we will see more multi-period portfolio optimization

Presenter Biography

Petter Kolm is the Deputy Director of the Mathematics in Finance Masters Program at the Courant Institute of Mathematical Sciences, New York University and a Partner at Kolm Financial Consulting. His research interests include quantitative trading strategies, delegated portfolio management, financial econometrics, risk management, and optimal portfolio strategies. Previously, Petter worked in the *Quantitative Strategies Group* at *Goldman Sachs Asset Management* where his responsibilities included researching and developing new quantitative investment strategies for the group's hedge fund. Petter coauthored the books *Financial Modeling of the Equity Market: From CAPM to Cointegration* (Wiley, 2006), *Trends in Quantitative Finance* (CFA Research Institute, 2006), and *Robust Portfolio Management and Optimization* (Wiley, 2007). He holds a Ph.D. in mathematics from Yale, an M.Phil. in applied mathematics from Royal Institute of Technology, and an M.S. in mathematics from ETH Zurich. Petter is a member of the editorial board of the *Journal of Portfolio Management*.

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Endnotes

[1] For further details, see Kolm and Maclin, "Algorithmic Trading ", (to appear in *Encyclopedia of Quantitative Finance*, edited by Rama Cont, John Wiley & Sons Ltd., 2009.

[2] Total costs are given by implementation shortfall plus commissions, where implementation shortfall is defined as the sum of (1) timing delay costs, and (2) market impact costs.

[3] Joint work with Lee Maclin.

[4] Huberman and Stanzl (2004) provide a no-arbitrage argument for this assumption.

[5] For further details, see Kolm and Peynetti, "Portfolio Optimization with Market Impact Costs", in preparation.

[6] Note that because of the increasing slopes of the linear segments and the goal of minimizing that term in the objective function, the optimizer will never set the decision variable corresponding to the second segment, $y_{2,i}$, to a number greater than 0 unless the decision variable corresponding to the first segment, $y_{1,i}$, is at its upper bound. Similarly, the optimizer would never set $y_{3,i}$, to a number greater than 0 unless both $y_{1,i}$, and $y_{2,i}$, are at their upper bounds. So, this set of constraints allows us to compute the total traded amount of asset i as $y_{1,i} + y_{2,i} + y_{3,i}$.

[7] Rote, "The Convergence Rate of the Sandwich Algorithm for Approximating Convex Functions," *Computing*, 48, pp 337-361, 1992.

[8] Ceria, Takriti, Tierens, and Sofianos, "Incorporating the Goldman Sachs Shortfall Model into Portfolio Optimization," presentation at Axioma's Breakfast Research Seminar Series, New York, 2008

[9] For further details, see Kolm, "Multi-Period Portfolio Optimization with Transaction Costs, Alpha Decay, and Constraints", in preparation.